# Marginally non-Gaussian Inverse Stochastic Modeling

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**Abstract** Recently, in the fields of hydrogeology and petroleum engineering, a big effort has been made in the development of techniques for the characterization of geological formations from sparse data. The aim of the latest developments is the generation of realizations of the geological architecture that display realistic patterns of variability, such as those observed in outcrops. This characterization is only possible using non-multiGaussian statistical models. Also recently, in these same fields, a big effort has been made in inverse modeling (or history matching) with emphasis in the generation of multiple realizations of the parameters that control flow and transport in the sub-surface. All of these realizations must be conditional on the measurements of the state variables, in the sense that the numerical solution of the state equation in these realizations predicts correctly the state at measurement locations. But, there is no efficient technique capable of generating non-multiGaussian realizations that are, at the same time, conditional to state-variable measurements. The objective of this work is to present such a technique through a fusion of Gaussian transformations and ensemble Kalman filtering.

## Introduction

The ensemble Kalman filter (EnKF) is widely used in hydrogeological inverse modeling and petroleum engineering history matching. It is well known that the Kalman filter is optimal for Gaussian processes with a linear state equation. The ensemble Kalman filter is quite robust to handle non-linear state equations, such as the ones controlling fluid flow and mass transport in the subsurface; however, it fails to handle non-Gaussian processes.

Two ways to approach the optimality conditions of the Kalman filter would be to find an alternative representation of the system based on a linear state equation (i.e., Chen, Oliver, & Zhang, 2009), or to transform the state variables into

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Gaussian ones (i.e., Sun, Morris, & Mohanty, 2009; Schöniger, Nowak, & Hendricks Franssen, 2012). We have worked on the second approach, but in contrast with previous works, this work focuses on transforming not only the non-Gaussian distributed state variables, but most importantly, the non-Gaussian distributed model parameters, i.e., the hydraulic conductivities. These transformations of parameters and state variables will make the relationship between the transformed variables even more nonlinear, but the Kalman filtering equations will be applied to Gaussian variables.

## Method

The normal-score ensemble Kalman filter discussed by Zhou, Gómez-Hernández, Hendricks Franssen, & Li (2011) is based on the standard ensemble Kalman filter. Let $\mathbf{x}$ be the augmented state vector of conductivities and piezometric heads. In the standard Kalman filter, the forecast step involves the transition of the augmented vector from time $t$-1 to time $t$,

$$\mathbf{x}_t = F(\mathbf{x}_{t-1})$$

in which both $\mathbf{x}_t$ and $\mathbf{x}_{t-1}$ are non-Gaussian variables, and $F$ represents the state equation. In the normal-score ensemble Kalman filter, the forecast and analysis is applied on the normal-score transform of the augmented state vector. Let $\Phi_t(\cdot)$ represent the normal-score transform at time $t$ so that

$$\mathbf{y}_t = \Phi_t(\mathbf{x}_t)$$

is a new state vector with all variables having a marginal Gaussian distribution with zero mean and unit variance. Similarly, we have a normally distributed state vector at time $t$-1, $\mathbf{y}_{t-1} = \Phi_t(\mathbf{x}_{t-1})$, therefore we can write

$$\mathbf{y}_t = \Phi_t(F(\Phi^{-1}_{t-1}(\mathbf{y}_{t-1}))),$$

that can be rewriten as

$$\mathbf{y}_t = \psi_t(\mathbf{y}_{t-1})$$

with $\psi_t = \Phi_t \cdot F \cdot \Phi^{-1}_{t-1}$. We have replaced the original non-linear transfer function $F(\cdot)$ by a new non-linear transfer function $\psi_t(\cdot)$ that takes as input a vector of Gaussian variables that propagates in time into a new vector that is also Gaussian. The analysis step is applied as in the standard EnKF to the new state vector.

It is clear that the normal-score transform only renders the variables marginally Gaussian, all higher-order moments may remain far from it.

We have transformed the standard EnKF into another one with a state equation that depends on time, since the normal-score transform must be recomputed at each time step.

The NS-EnKF has been tested in several highly non-Gaussian aquifers and has proven to perform extremely well, as long as there is enough conditioning information in the form of transient piezometric heads (see, for instance, Li, Zhou, Hendricks Franssen, & Gómez-Hernández, 2012).

## *Acknowledgments*

The authors gratefully acknowledge the financial support by the Spanish Ministry of Science and Innovation through project CGL2011-23295, and by the Universitat Politècnica de València through project PERFORA.

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