Use of Streamlines in Development and Application of a Linearized Reduced-Order Reservoir Model

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Abstract Reduced-order modeling procedures are very useful techniques where the simulation model must be run many times (i.e. optimization problems) or dimensions of the model are large. Therefore there is a significant need to reduce the computational requirements for the flow simulation model. This paper presents a localization method to construct a new reduced-order linear model for a multi-phase flow in underground formations which is highly nonlinear. The proposed model takes advantage of streamlines using information gleaned from streamline trajectories. Mathematical methods (e.g. singular value decomposition) are frequently used in reservoir model order reduction procedures. In this work streamlines are used as an alternative to these classical techniques to decide which grid blocks' state variables are mostly effective on each local models. Obviously the state variables are a complete description of a system. Therefore reservoir can be thoroughly described through state variables, related to the remaining streamlines of the cells. It is obvious that analyzing and controlling the behavior of a linear system is much easier than a nonlinear one. In this work, a novel linearization method in reservoir model is proposed. Dynamic variables of a reservoir such as pressure have local dependency, which means their values depend only on neighboring grids' values and have no dependency in value to farther grids. This statement appears to be physically reasonable as well, since it abides by the

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reservoir fluid flow equations. Therefore dynamic variables of any curve can be estimated by dynamic variables of neighboring curves' grid blocks. In the present work, streamlines play the role of the so-called curves and the linear relationship between neighboring streamlines is developed employing least square method. It is shown that the local models which are highly nonlinear in Cartesian or Radial coordinates, are linear in streamline based coordinates. This contributes to have a linear global model. The results of simulation on two synthetic cases verified the accuracy of the method.

Keywords: Linearization; Model Order Reduction; Streamline, Localization.

1. Introduction

Solving highly nonlinear unsteady reservoir fluid flow problems several times, rapidly, and iteratively is physically impossible. This is the case even by the aid of current computer-CPUs. One of the reasons behind this issue is that solving nonlinear equations typically involves a large matrix factorization at every time step of simulation. However analyzing and controlling the behavior of a linear system is much easier than a nonlinear one, since solving the linear form of flow equations requires only one matrix factorization. As such, applying linearization methods in solving complex reservoir fluid flow problem has been proposed recently in several studies. This work, presents a novel linearization method in reservoir model.

In [1-2] the fluid flow equations are expanded around previously simulated (saved) states and corresponding controls. Performing one or more high-fidelity training simulations is necessary in trajectory piecewise linear method (TPWL) [2]. Thus, in an uncertain model (i.e. in history matching process where the simulation of uncertain model needs to be run many times), the method cannot be useful and the linearized model would probably diverge in the course of history matching. In previous fluid flow linearization methods the coordination is not subject to any changes; however in the present paper, we show that the model which behaves quite nonlinearly in Cartesian or Radial coordinates behaves linearly in streamline-based coordinates.

In the control design of a fluid flow system, a substantial gain can be obtained using linearized model. In a two-phase flow mode, the number of states is twice the number of model grid-blocks. Thus control design, particularly online control implementation, is impractical due to the fact that the computational cost of optimal control design is a power function of system dimension (O [n3]), where n represents system dimension). Model order reduction is a way to overcome this problem [3].Projecting the equations onto a set of basis function spanning the flow solution space, is the central idea of previous works, e.g. combination of TPWL with Arnoldi reduced order models [4]. Nonlinear analogue circuits and micro machined devices are some examples of the application of this method. Also for this application, a truncated balanced realization algorithm has been combined with the TPWL order reduction approach [5]. Proper orthogonal decomposition (POD) is another approach in the reduced order model procedure [2].

In the previous works, nonlinearity effects were neglected as model order reduction was applied to a linearized model. The range of validity of linearized model is restricted to a small perturbation around a steady state. In this work, instead of using basis functions, state variables of grid blocks categorized into some groups and that ones which pass through streamlines are selected as state variables of the reduced order local models. These state variables are strongly correlated to observation data of each local model. Apparently, grid blocks that no streamline passes through them, have no effect on our approach on reduction of reservoir model.

This paper shows that state variables (water saturation, pressure) of each grid block has a strong correlation with neighboring grid-blocks' values, while the effect of so called correlation is less for farther grid-blocks. Therefore state variables of any trajectory lines which connect a source to a sink and pass through certain number of grid blocks can be approximated by state variables of neighboring trajectory lines passing through neighboring blocks. In the present work, streamlines play the role of the so-called trajectories. A linear relationship between neighboring streamlines is developed employing least square method.

In this study, we first summarize the governing equations for oil-water flow and develop a scheme to discretize them. Streamline simulation and least square method are then briefly described. In the next step, the development and application of the localization, reduction and linearization method in reservoir flow model is discussed. And finally some synthetic simulation cases are tested to verify the accuracy and efficiency of the proposed method.

2. Oil-Water Flow Equations and State Space Model

The fluid flow equations of reservoir are obtained by combining mass conservation equations with the Darcy's law [6].These partial differential equations do not have generally analytical solutions. Consequently, discretization of reservoir model is commonly performed using well-established techniques of finite element (FE), finite volume (FV), or finite differences (FD). The basic element of the spatial domain in a FD discretization is a grid-block, and the primary variables of the system, here p_o (oil pressure) and S_w (water saturation), are defined either at grid-nodes or at grid centers. By using FD technique, the PDE equations can be approximated for the grid block *ijk* as [7]:

$$V_{ijk}W_{ijk}\frac{dx_{ijk}}{dt}=T_{ijk}\begin{bmatrix}x_{i,j,k-1}\\x_{i,j-1,k}\\x_{i-1,j,k}\\x_{i,j,k}\\x_{i+1,j,k}\\x_{i,j+1,k}\\x_{i,j,k+1}\end{bmatrix}+G_{ijk}\begin{bmatrix}D_{i,j,k-1}\\D_{i,j-1,k}\\D_{i-1,j,k}\\D_{i,j,k}\\D_{i+1,j,k}\\D_{i,j+1,k}\\D_{i,j,k+1}\end{bmatrix}+V_{ijk}\tilde{q}_{ijk}. \tag{1}$$

where

$$x_{ijk}=\begin{bmatrix}p_{ijk}\\S_{ijk}\end{bmatrix}. \tag{2}$$

Stacking the above equations for all the grid-blocks *ijk* on top of each other yields a time-continuous *(generalized) state-space* formulation [7]:

$$VW\frac{dx}{dt}=Tx+Gd+V\tilde{q}. \tag{3}$$

where the state vector *'x'* consists of oil pressures and water saturations of each grid block, V is a diagonal matrix with entries that are functions of grid-block volume and fluid densities, W is a block diagonal matrix with entries being functions of compressibility, porosity and water saturation, T and G are sparse block matrices accommodating block-interface transmissibilities for oil and water, d is the depth-vector, and $\tilde{q}$ denotes the well flow-rates.

T matrix transmits the vector of previous step time state variables into the next one. The properties of the transmission matrix (spars block) enable us to split the large-scale state space model into the smaller ones which are independent from each other. It could be shown that the state variables of each grid block do not have necessarily a correlation with the state variables of all grid blocks, since there are some grid blocks which have zero coefficients in calculation of state variables of certain grid blocks. This fact is used in the development of the proposed method and shows that the state variables (i.e. pressure, saturation) along each streamline could be obtained by just knowing the value of the state variables of the adjacent streamline and the relation between these two lines.The relation between the lines is explained in section.

3. Streamline Simulation

Streamline simulation is a reservoir simulation technique by which the reservoir is split into series of one-dimensional trajectories, and the transport equations are solved along the trajectories. Each trajectory is called a streamline, and is computed by drawing the tangential line to the velocity vector, or orthogonal to pressure contours at each location. Instead of moving fluids cell-to-cell as in conventional simulators, a decoupling of the transport problem form the underlying 3D grid is made, such as the fluids can be transported more efficiently [24]. Under these conditions the time-steps are much larger compared to those in conventional finite difference simulators. Therefore streamline simulation is a fast technique (faster than finite difference simulator) to model large heterogeneous reservoirs with water injection as a mechanism for reservoir pressure maintenance [19-24].The technique is suited mainly for optimization techniques which require a number of simulations in sequence. The number of time steps is only a function of well events and also the changes in physics of fluid transport, and is independent of reservoir heterogeneity, reservoir geometrical properties and grid block size and orientation [20].

Due to its versatility, streamline simulation has been used in a number of applications, example screening and ranking geostatistical models, rapid assessment of production strategies, and upgridding and upscaling [21-24]. Other advantages of streamline simulators are reduction in grid-orientation effects and quantitative flow visualization [24]. The last mentioned attribute of streamlines is what we mostly look for, to develop the proxy, as the trajectories carry some fluid flow and petrophysical information along themselves.

4. Least Square Model Fitting

Least square method is one of basic methods of identification. Since the method is one of main bases of this study, the least square model fitting is summarized in this section.

The model relates an observed variable y_t (the regressand), to *p* explanatory variables (the regressor) u_{1t} to u_{pt}, and all are either known in advance, or observed. The model deals with only one unknown coefficient per each explanatory. Thus, if the regressor vector has *p* elements, coefficients are collected into p-vectors [10]:

$$u_t = [u_{1t} \quad u_{2t} \quad \ldots \quad u_{pt}]^T . \tag{4}$$

$$\theta = [\theta_1 \quad \theta_2 \quad \ldots \quad \theta_p]^T . \tag{5}$$

Then the model is:

$$y_t = f(u_t, \theta) + e_t \qquad t = 1,2,3,\ldots,N. \tag{6}$$

Where e_t accounts for observation and modeling error. The main objective of the method is finding the value $\hat{\theta}$ (function of θ) which minimizes S defined as follow:

$$S = \sum_{t=1}^{N} e_t^2 = \sum_{t=1}^{N} (y_t - f(u_t, \theta))^2. \tag{7}$$

In linear cases, $f(.,.)$ is linear in the unknown coefficients

$$y_t = u_t^T \theta + e_t \quad t = 1,2,3,...,N. \tag{8}$$

To make the algebra tidy, N-vectors Y and e are defined as a collection of all samples y_1 to y_N and e_1 to e_N respectively. The $N \times p$ matrix U is a collection of all the u_t vectors.

$$Y = U\theta + e. \tag{9}$$

and

$$S = e^T e \tag{10}$$

The value $\hat{\theta}$ which minimizes S is obtained by (11):

$$\hat{\theta} = [U^T U]^{-1} U^T Y. \tag{11}$$

To check that $\hat{\theta}$ gives a minimum value of S (not a maximum or saddle point), any small change $\delta\theta$ about $\hat{\theta}$ must increase S. If $U^T U$ is positive-definite, then $\delta\theta^T U^T U \delta\theta$ cannot be zero for any real non-zero $\delta\theta$ and also the existence of the inverse of $U^T U$ is guaranteed.

5. Development of the Method for Linearization and Reduction of Reservoir Model

5.1. Streamline-based Assisted Localization and Reservoir Model-reduction

A typical commercial reservoir simulator provides some dynamic outputs such as water cut, production rate, etc for a given reservoir and well condition as inputs. The schematic of a flow simulator is shown as below:

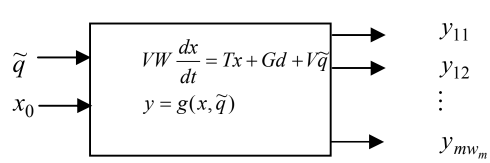

Figure 1- Schematic of a Flow Simulator

where y_{ij} is the j^{th} output of i^{th} well, and m is the number of wells. If the vector of outputs is decomposed into m vectors $z_i(i = 1,...,m)$, z_i being the output of the i^{th} well, then the system can be decomposed into m sub-systems. Each sub-system has its own region of influence, in a way that the region has the largest possible influence on the outputs of a certain well [14]. If the region of influence for each sub-system is defined, the model obtained from each sub-system does not have the complexity of the original system since there is a large group of state variables that has no effect on the outputs of a given sub-system. Therefore, each sub-system has a limited number of state variables and a simple state space model. There are certain ways to identify the regions of influence of each sub-system [15]. The use of streamline trajectories to identify these regions has proved to be useful in the past, especially in history matching problem which deals with large scale system. In [15] streamlines were used to identify the grid-blocks that affect the production response in a specific well. In this paper, the information gleaned from streamline trajectories is used to identify the effective grid-blocks for a specific sub-system, then it is possible to restrict the model of subsystem to these grid blocks. In fact, the large scale reservoir model is localized into a number of subsystems that is equal to the number of wells. Previously Johansen and Foss applied local model systems for diagnosis, modeling and control [16-18]. Any model will have a limited range of validity which may be restricted by the experimental conditions. A model is called "local model", when it has a range of validity that is smaller than the desired range of validity. Moreover, "operating regime" is a region in which local model is valid [16]. In this paper, local models are the models of each sub-system and the operating regime of each model is defined using streamline trajectories. Therefore, many state variables which have no influence on the observations are truncated from the proposed model, decomposing into the significantly small local models.

Construction of the proposed reservoir model has two main steps: (1) Decomposition of output vector of the reservoir model into a number of output vectors equal to the number of wells. Each new output vector belongs to a certain well and contains the outputs of the well. (2) Identification of the effective grid- blocks of each sub-system using streamline trajectories. Third, a local model structure must be developed for each operating regime. It is necessary to mention that if the local structures are linear in parameters, the global model will be linear in those parameters. Therefore, the parameters can be identified by standard system identification tools. In the next section, the local linear models will be developed using least square method.

5.2. Local Linear Model Structure Development

In the introduction section, the differences between a linear and non-linear system were mentioned in terms of behavior control, and it was recommended to use a linear system. Also the level of correlation between some certain grid-blocks (connected by a streamline trajectory) with the neighboring and farther grid-blocks were discussed. This section explains how to develop a linear relation between each pair of adjacent streamline trajectories. We show here that a non-linear model can be transformed to a linear one using streamline-based coordinates. To verify this approach, the simulation results of a synthetic case are brought in the next section.

A simulation of the full flow model must first be performed and the state variables of the system (saturation and pressure of all grid-blocks in two-phase flow) should be saved. The streamlines' configuration and distribution must be calculated in the next step using a streamline tracing algorithm, or any other streamline simulation commercial tools (i.e. 3DSL, FrontSim).

By eliminating some grid blocks that streamlines of a certain well do not pass through them, a few numbers of grid blocks remain that carry out the most significant state variables of the systems. Obviously the state variables represent a very good description of a system, therefore the state variables related to the remaining cells, can very well describe the desired operating regime of the reservoir. After eliminating excessive state variables, streamlines are sorted based on the proximity to each other .A value from 1 to N (N is the number of remaining streamlines, after elimination) is assigned to each streamline.

Having N regular lines, and knowing that they have a linear relationship, we need to figure out this relationship. Least square is a good approach to obtain the so called relationship, due to its linearity and having sample data in several time steps. The next part explains how to obtain the linear relationship.

5.2.1. Implementation of Least Square in Model Construction

First, some definitions are necessary to explain the method:

$S(i,j)$ =index of jth cell of ith streamline.

$Ns(i)$ =the number of cells that the ith streamline pass through

$Xp_i(t)$ = Pressure of the cell with index i, t is index of time.

$Xs_i(t)$ = Saturation of the cell with index i, t is index of time.

The regressand vector of i^{th} streamline is the state variables of grid-blocks that $(i+1)^{th}$ streamline passing through them. Similarly, the regressor vector of i^{th} streamline is state variables of grid blocks that the i^{th} streamline passing through them.

$$
\begin{aligned}
y_i(t) = \big[& Xp_{s(i+1,1)}(t) \quad Xp_{s(i+1,2)}(t) \quad \dots \quad Xp_{s(i+1,Ns(i+1))}(t) \\
& \dots \quad Xs_{s(i+1,1)}(t) \quad Xs_{s(i+1,2)}(t) \quad \dots \quad Xp_{s(i+1,Ns(i+1))}(t) \quad \big]^T. \\
u_i(t) = \big[& Xp_{s(i,1)}(t) \quad Xp_{s(i,2)}(t) \quad \dots \quad Xp_{s(i,Ns(i+1))}(t) \\
& Xs_{s(i,1)}(t) \quad Xs_{s(i,2)}(t) \quad \dots \quad Xp_{s(i,Ns(i))}(t) \big] \\
y_i(t) = {} & \theta_i u_i(t).
\end{aligned}
\tag{13}
$$

The problem is converted to a simple linear regression equation. Knowing the n sample times of the vectors at $(t_1,t_2,...,t_n)$, one of the best approach to solve the problem and obtain the optimum value of θ for the regression equation is least square method. θ_i for each streamline is obtained by least square method as follow:

$$Y_i = \begin{bmatrix} y_i^T(1) \\ y_i^T(2) \\ \vdots \\ y_i^T(n) \end{bmatrix}_{n\times Ns(i+1)} ,U_i^T = \begin{bmatrix} u_i^T(1) \\ u_i^T(2) \\ \vdots \\ u_i^T(n) \end{bmatrix}_{n\times Ns(i)} ,Y_i = U_i^T\theta_i. \tag{14}$$

$$\hat{\theta}_i = (U_i^T U_i)^{-1} U_i^T Y_i. \tag{15}$$

Consequently, if the value of state variables of grid blocks through which the first streamline is passing, are known during the time steps (the elements of $u_1(t)$ are known), the values of all state variables of all streamlines can be calculated. The linear local models can be simply replaced by simulator, thus the proposed method is very time consuming. The remaining issue is that elements of $u_i(.)$ are not completely independent of one another as they are the values of state variables of gird-blocks through which a certain streamline passes. Singular Value Decomposition (SVD) is coupled with the method to overcome this problem. The next section explains briefly the coupling approach of SVD with the proposed method.

5.2.2. Singular Value Decompostion Assisted in Model Construction

Matrix U_i can be transformed into the below form by singular value decomposition [11,12]:

$$U_i = WSV^T. \tag{16}$$

with $W^TW = UU^T = I_{Ns(i)}, V^TV = VV^T = I_n$ and S being $Ns(i)\times n$ with singular values of U_i distributed diagonally. Singular values of U_i are positive square roots of eigen values of $U_iU_i^T$. The columns of V are orthonormalized eigen vectors of $U_i^TU_i$ and the columns of W are orthonormalized eigen vectors of $U_iU_i^T$. The rank of U_i equals the number of nonzero singular values. If the rank of U_i is r, the first r columns of W are orthonormal basis of the range space of U_i. Singular values of stable system indicate the respective state energy of the system [2]. Therefore, reduced order can be directly determined by examining the system singular values. If $s_1,...,s_n$ are singular values of U_i in decreasing order, singular values with small amount can be removed. The ratio of remaining energy to total energy is calculated as follows:

$$\frac{\sum_{i=1}^{l} s_i^2}{\sum_{i=1}^{n} s_i^2}. \tag{17}$$

Transfer matrix φ will contain only first l columns of W. Then we have:

$$U_i' = \varphi_i^T U_i. \tag{18}$$

$$\hat{\theta}_i' = (U_i'^T U_i')^{-1} U_i'^T Y_i. \tag{19}$$

$$\hat{\theta}_i = \varphi_i \hat{\theta}_i'. \tag{20}$$

Two problems are solved using SVD: First, singularity of inverse problem in calculating θ_i is solved by transforming the space into the space which is spanned by vectors related to largest singular values. Secondly, the computational load is significantly reduced due to dimension reduction of U_i to U_i'.

5.2.3. Wave Advanced Model Construction

By defining new state vector, *X(k),* which has state variables of gird blocks passing through k[th] streamline for each operating regime, Wave Advanced Model (WAM) is constructed as below:

$$X(k+1) = A(k)X(k). \tag{21}$$

where

$$A(k) = \theta_k, X(k+1) = y_k, X(k) = u_k. \tag{22}$$

It can be seen that the size of state vector is varied by the number of cells passing through each streamline. The linearity of local models along streamlines contributes to having linear global model along streamlines. This approach can be used in controller design or other field of reservoir engineering such as history matching problem.

Remark: Obviously the state variables are a complete description of a system. Therefore reservoir can be thoroughly described through reduced state variables. Moreover, the value of outputs can be obtained by using an intelligent proxy such as fuzzy systems where their inputs are reduced state variables and its outputs are output of the reservoir model.

This model has some important advantages as below:

1- This model scans space instead of time which is following by time consuming. This is mainly because the large state vector of reservoir model decomposed into a number of state vectors and at each step just state variables of this new state vector is updated. The localization procedure is very simple and can be done by categorizing the outputs by their own wells.

2- As the proposed model contains the relation between state variables of two consecutive streamlines, this model has more information than the standard state space model. Therefore, the unobservability of the model is decreased by using the proposed model. This result can be very useful in history matching problem.
3- According to the one dimensionality and linearity of the state vectors in equation (22), all linear state estimator and controller can be used. For instance, the classical Kalman filter can be used by some changes in its formulation because of the size variation of state vectors.
4- In addition to the state vector decomposition, the state variables are reduced to the effective ones using streamline trajectories. Therefore, the error of model reduction can be negligible.

6. Example

Case I: In this example we consider a two- dimensional reservoir, which is discredited by 25×25 grids of 20^m length along each horizon. The model contains two production wells and a couple injector wells. The entire configuration of streamlines' paths of the model is illustrated in Figure 2. It can be seen that the model can be categorized into two groups: (1) the production well *P1* and the injection well *I2*, (2) the production well *P2* and the injection well *I*1. Each group has 144 streamlines. We start from the first right streamline to scan all streamlines of first group and obtain the the linear relationship between these lines. For instance, the relation between state variables of second and first streamlines is obtained by least square method. Therefore, the state variables of second streamline can be obtained by knowing the state variables of first line and the relation between these two lines. Finally, we scan whole of first region of reservoir. All of these tasks are done for the second region. After obtaining the linear relations between all of consecutive streamlines, the obtained model is tested by estimating the state variables of all streamlines by knowing just the values of the first right and left ones and the relations which obtained in the previous step. In Figure 3-5 the value of the pressure of the proposed method and simulator are compared in three different step times, which show that the estimated values are well matched to the real ones. The value of saturations equal to 0.25 and the estimated ones have this exact value.

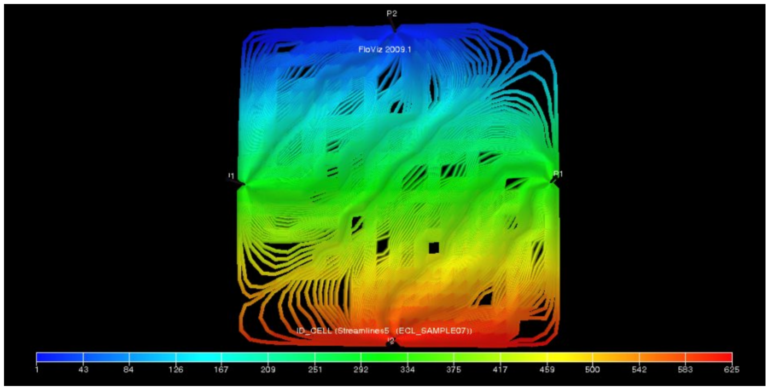

Figure 2- Model Case I

In order to compare quality of proposed method, root mean square (RMS) error is used as the criterion, which is defined as

$$\sqrt{\frac{1}{n}\sum_{j=1}^{n}(\hat{X}_j - X_j)^2} \tag{23}$$

where n is number of reduced cells (in this case it equals to 548) , $\hat{X}_j$ is the estimation of state variables in j^{th} cell and X_j is the value of state variables obtained by simulator in j^{th} cell. In Table I, RMS of state variables in several time steps can be seen. The results verify the accuracy of the claim about the linear relations between two consecutive streamlines.

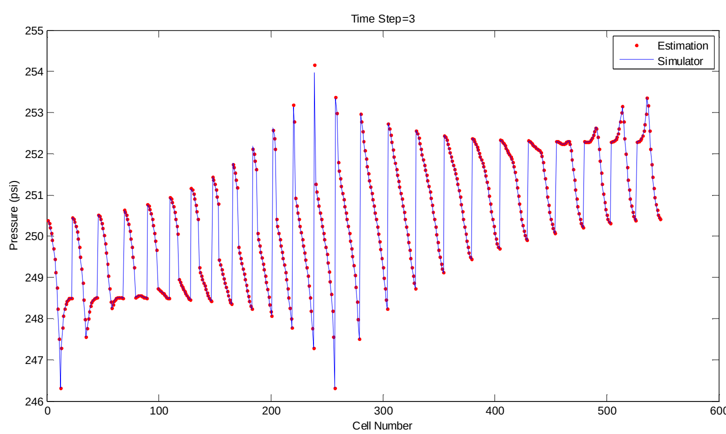

Figure 3- Comparison between Estimated Pressure of Reduced Cells by Proposed Model & Values of Simulator, Time Step=3

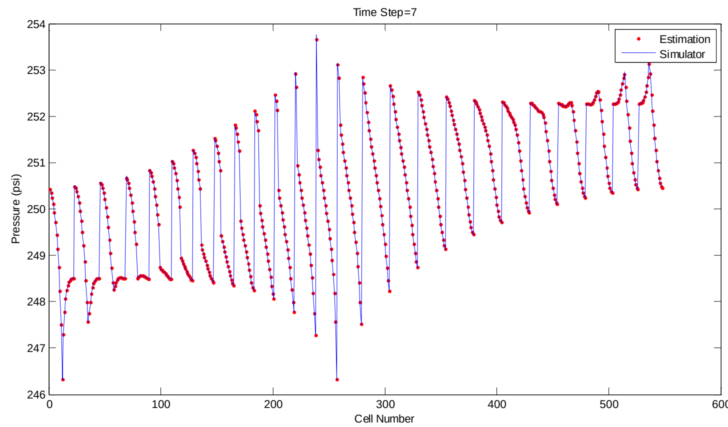

Figure 4- Comparison between Estimated Pressure of Reduced Cells by Proposed Model & Values of Simulator, Time Step=7

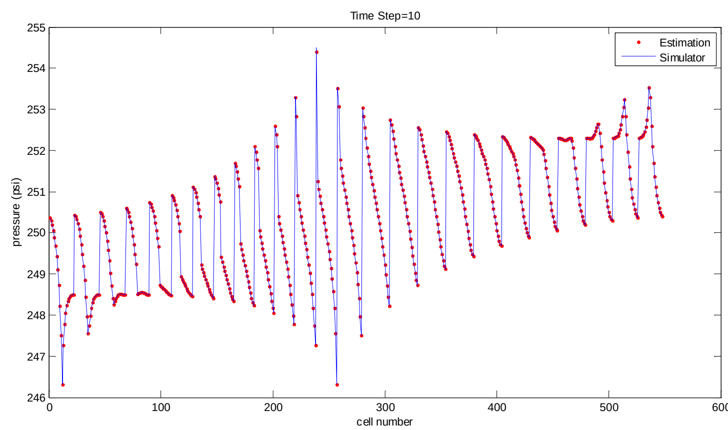

Figure 5- Comparison between Estimated Pressure of Reduced Cells by Proposed Model & Values of Simulator, Time Step=10

Table I- RMSE of state variables of the proposed method

Time-step:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
RMSE of Pressure	5×10^{-10}	2×10^{-10}	3×10^{-10}	2×10^{-10}	7×10^{-10}	2×10^{-10}	2×10^{-10}	5×10^{-10}	5×10^{-10}	5×10^{-10}
RMSE of Saturation	4×10^{-30}	1×10^{-30}	4×10^{-30}	1×10^{-30}	4×10^{-30}	0	0	0	1×10^{-30}	1×10^{-30}

Case II: The model and the entire configuration of streamlines' paths are illustrated in Figure 6. The reservoir model can be categorized into four groups of streamlines. Then the coefficients of linear state space local model, WAM, are computed by least square method assisted SVD for each group. In Table II, NRMS of state variables related to some random streamlines are mentioned, quantitatively. This shows that the proposed model is fairly accurate. Comparison between two models' pressure and saturation of a random cell are shown as an example in Figure 7.

Table II- NRMS of state variables related to streamlines

Number of Streamline	2	10	19	25	31	36
NRMSE	0.002	0.002	0.013	0.014	0.018	0.002

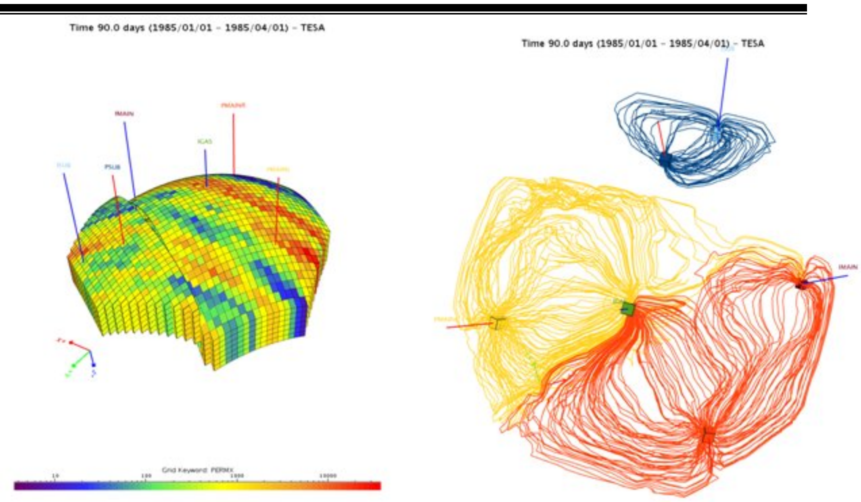

Figure 6- Model II (Permeability distribution on 3D structural model), Right: Streamline configuration of well pairs.

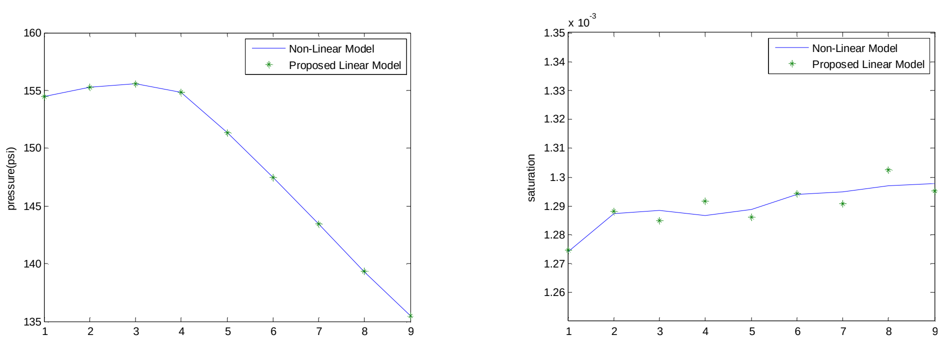

Figure 7- Matching result for simulation data and proposed linearized method for first grid of second streamline's pressure (left) and saturation (right).

7. Conclusion

A new reduced-order linear localized model (streamline-based proxy) was presented for a highly nonlinear multi-phase flow in underground formations. The proposed model took advantage of streamlines using information gleaned from streamline trajectories.

In this work localization method was used to categorize the output vector of reservoir model and streamlines were used as an alternative to mathematical techniques to identify the effective region of each local model. The current work presents a novel linearization method in reservoir modeling. It was shown that the model which is highly nonlinear in Cartesian or Radial coordinates is linear in streamline-based coordinates, due to the linearity of local models along streamlines. The results of simulation on a synthetic case verified the accuracy of the method. The method can be a very useful technique where the simulation model must be run many times (i.e. optimization problems) or where the dimensions of the model are large or the original model has a high uncertainty (i.e. history matching problem). The method had significant computational reduction compared to current reservoir simulators, and can be used a proxy model for optimization techniques.

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