Seismic AVO inversion using the closed skew-normal random field

Kjartan Rimstad and Henning Omre

Abstract Skewness is often present in a wide range of geostatistical problems, and modeling it in the spatial context remains a challenging problem. In this article the multivariate closed skew-normal distribution is considered, which is a generalization of the traditional normal distribution. The closed skew-normal distribution is used in a predictive setting on real seismic data from the Sleipner field in the North Sea. Model parameters are estimated by maximum likelihood, and the predictive distribution is estimated by a Metropolis-Hastings algorithm.

1 Introduction

The multivariate closed skew-normal (CSN) distribution is defined in González-Farías et al. (2004), as a generalization of the multivariate normal distribution. Let

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{pmatrix} \stackrel{d}{=} \mathrm{N}_{p+q}\left[\begin{pmatrix} \mu \\ \nu \end{pmatrix}, \begin{pmatrix} \Sigma & -\Sigma \mathsf{D} \\ -\mathsf{D}^T\Sigma & \Delta + \mathsf{D}^T \Sigma \mathsf{D} \end{pmatrix} \right], \tag{1}$$

where $\mathbf{U} \in \mathbb{R}^{p+q}$, $\mathbf{U}_1, \mu \in \mathbb{R}^p$, $\mathbf{U}_2, \nu \in \mathbb{R}^q$, $\Sigma \in \mathbb{R}^{p\times p}$, $\Delta \in \mathbb{R}^{q\times q}$, $\mathsf{D} \in \mathbb{R}^{p\times q}$, $\stackrel{d}{=}$ denotes equality in distribution, T denotes matrix transpose, and $\mathrm{N}_n(\mu, \Sigma)$ denotes the n-dimensional multivariate normal distribution with expectation vector μ and covariance matrix Σ. Then $\mathbf{X} \stackrel{d}{=} [\mathbf{U}_1 \mid \mathbf{U}_2 \leq \mathbf{0}]$ is defined to be CSN distributed, denoted $\mathrm{CSN}_{p,q}(\mu, \Sigma, \mathsf{D}, \nu, \Delta)$. Here the notation $\mathbf{U}_2 \leq \mathbf{0}$ corresponds to all elements in $\mathbf{U}_2$ being jointly less than zero. The corresponding probability density function

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(pdf) of the CSN distribution is

$$f_{p,q}(\mathbf{x};\mu,\Sigma,\mathsf{D},\nu,\Delta) = \phi_p(\mathbf{x};\mu,\Sigma)\,\frac{\Phi_q(\mathbf{0};\nu-\mathsf{D}(\mathbf{x}-\mu),\Delta)}{\Phi_q(\mathbf{0};\nu,\Delta+\mathsf{D}^T\Sigma\mathsf{D})}, \tag{2}$$

where $\phi_\cdot(\cdot;\cdot,\cdot)$ is the normal distribution pdf and $\Phi_\cdot(\cdot;\cdot,\cdot)$ is the normal cumulative distribution function (cdf). For D being a matrix of zeros, the CSN projects into the multivariate normal distribution. The class of CSN distributions inherits important properties from the multivariate normal distribution, such as being closed under marginalization, conditioning, and linear transformations (González-Farías et al. 2004).

Let $\left\{Z(\mathbf{s}) : \mathbf{s} \in \mathscr{D} \subseteq \mathbb{R}^d\right\}$ be a real-valued random field, where $\mathscr{D}$ is a finite domain in $\mathbb{R}^d$ and $\mathbf{s} \in \mathbb{R}^d$ is a generic location in $\mathscr{D}$. Then the random field $\{Z(\mathbf{s}) : \mathbf{s} \in \mathscr{D}\}$ is a Gaussian random field if for all configurations of points $(\mathbf{s}_1,\ldots,\mathbf{s}_n)$ and all $n > 0$ the pdf of $\mathbf{Z} = [Z(\mathbf{s}_1),\ldots,Z(\mathbf{s}_n)]^T$ is multivariate normal.

In Allard & Naveau (2007) a CSN random field is defined by first specifying a discretization $\mathbf{s}' = (\mathbf{s}'_1,\ldots,\mathbf{s}'_q)$, fixing q, and $\mathbf{U}_2 = \left[U_2(\mathbf{s}'_1),\ldots,U_2(\mathbf{s}'_q)\right]$. Then the associated CSN random field is defined as

$$\{X(\mathbf{s}) = [U_1(\mathbf{s}) \mid \mathbf{U}_2 \leq \mathbf{0}] : \mathbf{s} \in \mathscr{D}\}, \tag{3}$$

if for all configurations of points $(\mathbf{s}_1,\ldots,\mathbf{s}_p)$ and all $p > 0$ the pdf of $\mathbf{X} = [X(\mathbf{s}_1),$ $\ldots, X(\mathbf{s}_p)]^T$ is CSN distributed.

2 Seismic inversion

Inversion of seismic amplitude-versus-offset (AVO) data into elastic material properties in the subsurface is a major challenge in modeling of petroleum reservoirs. We consider AVO data from the Sleipner Øst field in the North Sea, and we have seismic AVO data from a 2D profile and observations of the elastic material properties from one well. In Buland & Omre (2003) the seismic inversion is casted in a Bayesian setting with Gaussian prior and likelihood densities. Karimi et al. (2010) consider a CSN model used on a 1D profile along one well and they use a pseudo-likelihood approach for parameter estimation. In this study we consider a 2D profile of the seismic data and we use the CSN random field as a prior model for the random field.

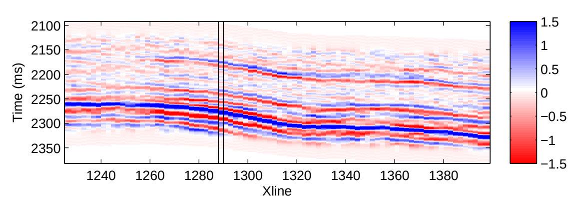

Fig. 1 Seismic amplitude data for 31°. The well is marked at around trace 1290.

The data **d** represent angle-dependent seismic AVO data for three angles [12°, 22°, 31°], and the data for 31° are displayed in Figure 1. We have observations every 4ms in the vertical direction and every 2 xline in the horizontal direction, and the dimension of the seismic data is $125 \times 88 \times 3 = 33\,000$.

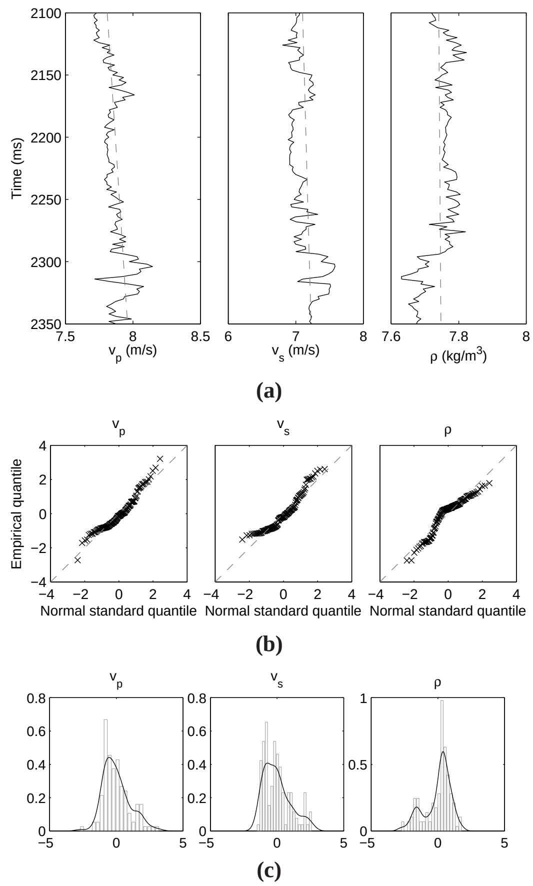

Fig. 2: Well observations. (a) Elastic properties in well with a dashed linear estimated trend, (b) quantile-quantile plot of empirical quantiles from data (residuals after removed linear trends) versus theoretical quantiles from normal distribution, (c) histogram and density plot of residuals.

The variable of interest **m** represents the logarithm of the elastic material properties of the subsurface. These elastic material properties are pressure-wave velocity, shear-wave velocity, and density, which also are observed along the well trace, displayed in Figure 2. The dimension of **m** is $p = 125 \times 88 \times 3 = 33\,000$. The elastic material properties are assumed to have linear trends, which are estimated in advance as illustrated in Figure 2(a). The residuals are displayed in Figure 2(b) and

2(c) in quantile-quantile plots and histograms, respectively, and all the elastic material property variables have some skewness in the empirical distributions.

The link between the observations and elastic material properties, termed seismic likelihood model $p(\mathbf{d} \mid \mathbf{m})$, is defined by a weak-contrast, convolutional, linearized Zoeppritz model (Buland & Omre 2003). The convolutional forward model is defined by $\mathsf{G} = \mathsf{WAD}$, where W is a convolutional matrix defined by wavelets given by the data owner, A is a matrix of angle-dependent weak contrast Aki-Richards coefficients (Aki & Richards 1980), and D is a differential matrix which calculates contrasts. The model is $\mathbf{d} = \mathsf{G}\mathbf{m} + \mathbf{e}$, where $\mathbf{e}$ is assumed to be a Gaussian error term with zero mean and covariance matrix Ω. The covariance matrix Ω is parameterized with a covariance structure that describes the relation between observation error for different angles, and a horizontal and a vertical range parameter. The likelihood is

$$p(\mathbf{d} \mid \mathbf{m}) = N(\mathsf{G}\mathbf{m}, \Omega). \tag{4}$$

The objective is to estimate the elastic parameters $\mathbf{m}$, from the observed seismic AVO data $\mathbf{d}$. We cast this inversion in a Bayesian setting; hence the posterior model is the objective

$$p(\mathbf{m} \mid \mathbf{d}) = \text{const} \times p(\mathbf{d} \mid \mathbf{m})\, p(\mathbf{m}), \tag{5}$$

where const is a normalizing constant and $p(\mathbf{m})$ is the prior model of $\mathbf{m}$. The prior model is defined to be a CSN random field. We choose to use a parsimonious model with few parameters such that we are able to estimate the parameters, but the model should be sufficiently flexible such that the model is able to describe different levels of skewness. We use $p = q$ and follow Allard & Naveau (2007) and let $\nu = \mathbf{0}$, then the prior model is

$$p(\mathbf{m}) = \text{CSN}_{p,p}(\mu, \Sigma, \mathsf{D}, \mathbf{0}, \Delta). \tag{6}$$

In the prior model μ is parameterized with one location parameter for each elastic material property. The covariance matrix Σ is parameterized with a covariance structure that describes the relation between the different elastic properties, and a horizontal and a vertical range parameter. The skewness parameter D is parameterized as a diagonal matrix such that each of the three elastic material parameter has the same parameter for the degree of skewness. The last parameter Δ is an identity matrix.

The model parameters in the prior and likelihood models, denoted θ, are estimated by maximum likelihood given well observations $\mathbf{m}_w^0$ and seismic data $\mathbf{d}$. Parameter estimation for CSN random fields poses challenging numerical problems, since multivariate normal cdfs are difficult to evaluate. We use importance sampling to estimate the multivariate normal cdfs (Genz 1992). By using the same set of uniform random variables for each likelihood function evaluation we ensure that the approximated likelihood is smooth; thus we are able to use standard optimization routines.

Given the estimated model parameters $\hat{\theta}$ the predictive distribution of interest $p(\mathbf{m} \mid \mathbf{d}, \hat{\theta})$ is also a CSN random field due to the closure properties of the CSN distribution. We use a block Metropolis Hastings algorithm to estimate the predictive distribution, where the algorithm is a block version of the Metropolis Hastings (MH) algorithm presented in Robert (1995) with proposal distributions from Genz (1992).

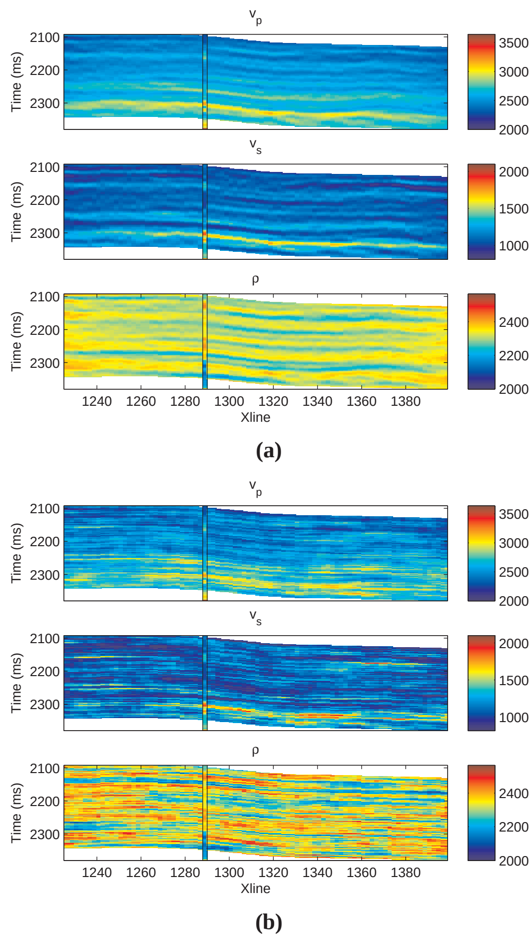

Fig. 3: Posterior median (a) and simulated samples from the posterior distribution (b), the well log is marked around trace 1290 and the values from the well log are plotted.

The posterior distribution is estimated by sampling 10 000 samples using the MH algorithm. Figure 3(a) displays the posterior median. The predictions for the density ρ have less precision than the predictions for v_p and v_s, but this is expected from the geophysics model since there is less information about ρ in the data. The well logs do not stick out significantly in the realizations from the posterior distributions in Figure 3(b).

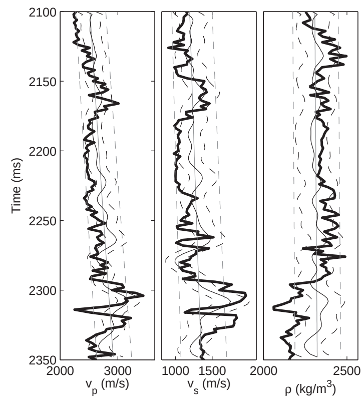

Fig. 4: Well predictions. Bold black solid is well observations, thin black solid is posterior median, dashed black is posterior 80% confidence interval, thin gray solid is posterior median, dashed gray is posterior 80% confidence interval.

Figure 4 displays the well log data $\mathbf{m}_w^o$ and the well predictions from the seismic data given by $p(\mathbf{m}_w \mid \mathbf{d}, \hat{\theta})$. The bold black solid lines are well observations, the thin black solid lines are posterior medians, dashed black lines are posterior 80% confidence intervals, the thin gray solid lines are prior medians, and the dashed gray lines are prior 80% confidence intervals. The predictions match well observations reasonably.

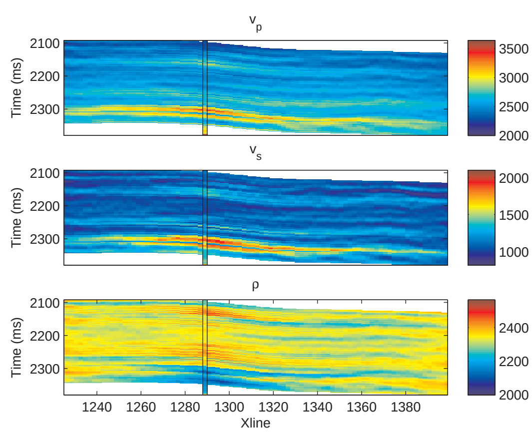

Fig. 5: Posterior median conditioned on the well observations.

In Figure 5 the well observations $\mathbf{m}_w^o$ are also used as data in the prediction, i.e. we use $p(\mathbf{m} \mid \mathbf{d}, \mathbf{m}_w^o, \hat{\boldsymbol{\theta}})$ as predictive distribution, which again is a CSN distribution due to the closure properties. These predictions gain higher resolution around the well, and the predictions are similar to the predictions without the well observations in Figure 3.

3 Conclusion

We have used the CSN random field model in a high dimensional predictive setting for a seismic inversion case with seismic AVO data and well observations. Model parameters are estimated by maximum likelihood, and the predictive distribution is estimated by a Metropolis-Hastings algorithm.

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