Wavelet estimation in seismic convolved hidden Markov models

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Abstract Inversion of seismic AVO-data is an important part of reservoir evaluation. These data are convolved but the convolution kernel and the associated error-variances are largely unknown. We aim at estimating these model parameters without using calibration observations in wells. This constitutes the first step in socalled blind deconvolution. We solve the seismic inverse problem in a Bayesian setting and perform the associated model parameter estimation by an approximate marginal maximum likelihood method. A small test study indicate that bell-shaped wavelets with smooth edges are identified well, even for approximations of low orders.

1 Introduction

In this study, we do parameter estimation by blind deconvolution for the seismic inverse problem. This is of importance in exploration and development of petroleum reservoirs. In seismic exploration, the seismic data is registered as a spatial convolution of the true physical properties in the subsurface Lithology-Fluid (LF) layers. Inference on the LF-classes based on the convolved seismic observations thus pose a deconvolution problem, which we address by a Bayesian approach, as presented in [Larsen et al. (2006)]. In particular, we focus on estimation of the wavelet parameters causing convolution effects. The full study is presented in [Lindberg and Omre (2012)].

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2 Model description

In a Bayesian framework, the solution to our inverse problem is represented by the posterior model which is defined from a prior model and a likelihood model, see [Larsen et al. (2006)]. The prior model is defined for the discrete LF field $\mathbf{x}$. The likelihood model defines the relationship of the seismic observations $\mathbf{d}$ conditioned on the LF field $\mathbf{x}$. We assume that the observations are registrered as a convolution of elementwise physical properties $\mathbf{r}$. As there are two unobserved levels, $\mathbf{x}$ and $\mathbf{r}$, where $\mathbf{x}$ has a Markov property and $\mathbf{r}$ is captured by convolution, we term the full model a convolved two-level hidden Markov model. The variables dependencies are displayed in Fig.1.

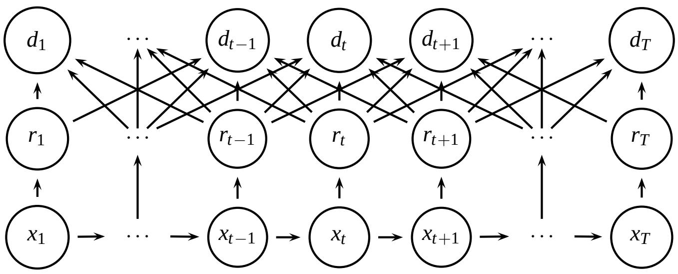


Fig. 1 Directed acyclic graph (DAG) of the convolved two-level HMM.

We assume that the underlying categorical variables follow a stationary first-order Markov prior model

$$p(\mathbf{x}) = \Pi_{t=1}^{T} p(x_t|x_{t-1})$$

where $x_t \in \{1,\dots,M\}$, and $p(x_t|x_{t-1})$ is defined from a transition matrix $\mathbf{P}$. We let $p(x_1|x_0) = p(x_1)$ for notational ease. We notice the first-order Markov dependencies in $\mathbf{x}$ in Fig.1.

The likelihood model is split into a response likelihood model and an acquisition likelihood model. The response likelihood model represent conditional independent rock physics properties which we assume to be Gaussian

$$p(\mathbf{r}|\mathbf{x}) = \Pi_{t=1}^{T} p(r_t|x_t) \qquad , \qquad p(r_t|x_t) = N(\eta_{x_t}, \sigma_{x_t}^2)$$

The acquisition likelihood model captures the convolution effect with Gaussian white noise

$$p(\mathbf{d}|\mathbf{r}) = N(\mathbf{W}\mathbf{r}, \sigma_d^2\mathbf{I})$$

Here $\mathbf{W}$ is a convolution matrix, with rows denoted as wavelets, $\mathbf{w}$. We assume the wavelets to be stationary and parametrized by a discretized symmetric beta model, $b(\alpha,\beta)$, where α is a shape parameter and β a discrete width parameter. The Beta model captures different wavelet shapes, see Fig.2. Hence each registration in $\mathbf{d}$ is a weighted sum of the elements in $\mathbf{r}$, with weights given by the wavelet $\mathbf{w}$. We notice the convolution effect of the response variables $\mathbf{r}$ in the observations $\mathbf{d}$ in Fig.1.

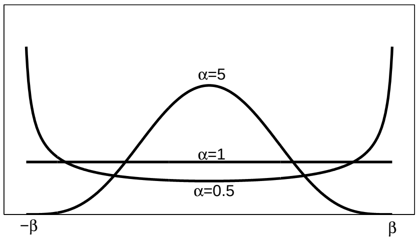


Fig. 2 Continous symmetric Beta model, $b(\alpha,\beta)$.

The posterior model is defined from the prior model and the likelihood model

$$p(\mathbf{x}|\mathbf{d}) = C \times p(\mathbf{x}) \times \int_{\mathbf{r}} p(\mathbf{d}|\mathbf{r})p(\mathbf{r}|\mathbf{x})d\mathbf{r}$$

Here C is a normalizing constant, which is infeasible to compute in practice as one need to sum over all M^T combinations of $\mathbf{x}$. In order to compute the full posterior model by the recursive Forward-Backward (FB) algorithm, the likelihood model need to be on factorizable form, see [Baum et al. (1970)]. We choose a kth order likelihood approximation according to [Rimstad and Omre (2012)],

$$\hat{p}^{(k)}\left(d_t^{(k)}|x_t^{(k)}\right) = \int \frac{p_*\left(r_t^{(k)}\middle|\mathbf{d}\right)}{p_*\left(r_t^{(k)}\right)} p\left(r_t^{(k)}\middle|x_t^{(k)}\right) dr_t^{(k)} \quad , \tag{1}$$

Here, $x_t^{(k)} = (x_{t-k+1},\dots,x_t)$ is a kth order LF-state, and similarly for $r_t^{(k)}$ and $d_t^{(k)}$. The functions $p_*(\cdot)$ are Gaussian approximations with analytically tractable parameters. A kth order approximate posterior model on factorizable form is thus obtained by

$$\hat{p}^{(k)}(\mathbf{x}|\mathbf{d}) = \Pi_{t=k}^T q^{(k)}\left(x_t^{(k)}|\mathbf{d}\right)$$

where

$$q^{(k)}(x_t^{(k)}|\mathbf{d}) = \begin{cases} C_k \times \Pi_{i=1}^k p(x_i|x_{i-1}) \cdot \hat{p}^{(i)}\left(d_i^{(i)}|x_i^{(i)}\right)^{1/k} & t = k \\ C_t \times p(x_t|x_{t-1}) \cdot \hat{p}^{(k)}\left(d_t^{(k)}|x_t^{(k)}\right)^{1/k} & t = k+1,\dots,T-1 \\ C_T \times p(x_T|x_{T-1}) \cdot \Pi_{i=1}^k \hat{p}^{(i)}\left(d_T^{(i)}|x_T^{(i)}\right)^{1/k} & t = T \end{cases} \tag{2}$$

Higher order approximations include more of the spatial dependencies caused by convolution, and should thus perform better, see [Rimstad and Omre (2012)].

3 Parameter estimation

We estimate the acquisition likelihood model parameters, $\theta_d = (\alpha, \beta, \sigma_d^2)$ which define the wavelet $\mathbf{w}$ based on the observed data only. This constitutes a blind deconvolution problem, and we use an approximate maximum likelihood estimation method. With an approximate likelihood by Exp.(1), it can be shown that a kth order approximate marginal log-likelihood can be computed by

$$\log\left(\hat{p}(\mathbf{d};\theta_d)\right) = -\Sigma_{t=k}^{T} \log\left(C_t\right) \tag{3}$$

see [Lindberg and Omre (2012)]. Here, C_t are the normalizing constants in Exp.(2) which computation requires $O(M^{k+1})$ operations. The approximate maximum likelihood is thus an optimization problem over a $O(TM^{k+1})$ function, as we need to run the FB algorithm for each evaluation of Exp.(3).

4 Example

The synthetic test data in this example are presented in Fig.3. In this small example we consider three possible LF-classes, represented by the colors white, grey and black in Fig.3. The prior model parameters used are

$$\mathbf{P} = \begin{pmatrix} 0.50 & 0.50 & 0 \\ 0.33 & 0.34 & 0.33 \\ 0 & 0.50 & 0.50 \end{pmatrix}$$

and response likelihood model parameters $\eta_{x_t} \in \{-2, 0, 3\}$ and $\sigma_{x_t} = 0.7$. In the test, we use two wavelet forms with associated observations. The observation $\mathbf{d}_1$ is simulated using a discretized Gaussian, $N(0,1)$, wavelet and the observation $\mathbf{d}_2$ using a Beta, $b(1.000, 6)$, wavelet. The two wavelets are displayed in red in Fig.4.

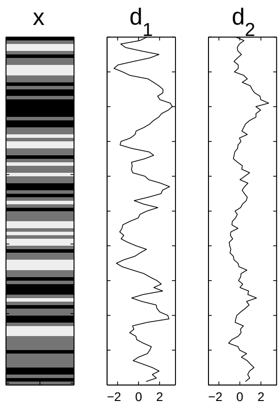


Fig. 3 Synthetic data in example. The LF field, $\mathbf{x}$, with three possible LF classes, the observation $\mathbf{d}_1$ simulated using a Gaussian, $N(0,1)$, wavelet and the observation $\mathbf{d}_2$ using a Beta, $b(1.000, 6)$, wavelet.

We have estimated the wavelet assuming a Beta shape and the acquisition noise variance parameter. The parameter estimates are presented in Table 1 and 2 for the observations $\mathbf{d}_1$ and $\mathbf{d}_2$ respectively. The corresponding estimated wavelets are displayed in Fig.4 compared to the true wavelets. The noise variance parameter is slightly overestimated for $\mathbf{d}_1$ and slightly underestimated for $\mathbf{d}_2$. Both wavelet estimates for $\mathbf{d}_1$ resemble the true wavelet very well. Notice how different parameter sets (α, β) may essentialy return the same Beta wavelet. For $\mathbf{d}_2$, the wavelet estimates differ significantly from the true wavelet. This might be due to the uniform wavelets sharp edges. We notice that the estimates do not improve significantly when we increase the order of the approximation from two to three.

Table 1 Parameter estimation results for $\mathbf{d}_1$

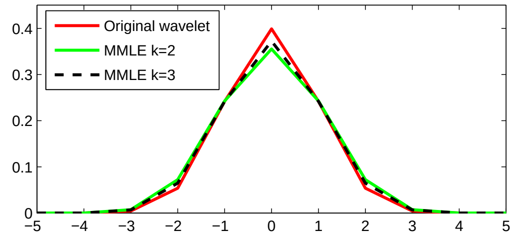
Parameter	Approximate MMLE		True value
	$k=2$	$k=3$	
α	14.5185	52.9414	$\sim$12.75
β	5	10	$\sim$4
σ_d	0.3652	0.3435	0.3000

Table 2 Parameter estimation results for $\mathbf{d}_2$

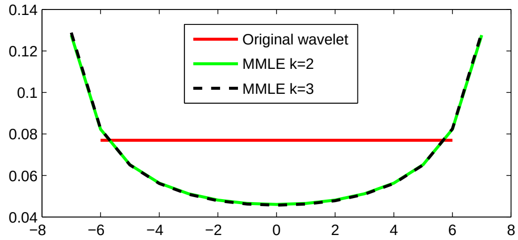
Parameter	Approximate MMLE		True value
	$k=2$	$k=3$	
α	0.2965	0.2851	1.0000
β	7	7	6
σ_d	0.2313	0.2440	0.3000

5 Discussion and conclusion

In this study, blind parameter estimation is performed for a convolved seismic inverse problem in a Bayesian setting. The work is inspired by seismic inversion of AVO-data into lithology/fluid (LF) classes. Wavelet and convolution noise estimation by a maximum likelihood method is performed in a small test study, computed by an approximation of the convolved likelihood model. Results indicate that bell-shaped wavelets are estimated well while uniform wavelets seem harder to recognize. There seem to be small improvements in the parameter estimates for higher order likelihood approximations. A lower order approximation thus provide a well trade-off between CPU-time required and estimation accuracy. The methodology is presented for a seismic trace in one dimension, but may easily be extended to higher dimensions by estimating parameters along each vertical trace of a field.



(a) $\mathbf{d}_1$



(b) $\mathbf{d}_2$

Fig. 4 Estimated and true wavelets for the two observed profiles. The original wavelet is displayed in red while the estimated wavelets for approximation order $k = 2$ and $k = 3$ are displayed in green and dotted black respectively.

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References

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