Spatial Prediction Using Minimun/Maximun Autocorrelation Factors and Multigaussian Kriging: Case Study in Mining Industry

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Abstract This paper deals with the joint estimation of a set of coregionalized variables, based on a transformation of the variables into minimum/maximum autocorrelation factors, assumed mutually independent, followed by multigaussian kriging in order to separately estimate the local distributions of these factors. The proposed approach is applied to one case study and the results are compared with the estimations by kriging and cokriging. The case study deals with the estimation of the total and soluble copper grades in a Chilean deposit. The estimation based on minimum/maximum autocorrelation factors shows a significant improvement in the reproduction of the relationships between the variables, in particular concerning the order relationship between total and soluble copper grades, while it considerably reduces CPU time with respect to the co-simulation of the variables.

1 Introduction

There is an increasing requirement in the mining industry to jointly estimate regionalised variables such as abundances of mineral species, total and recoverable

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copper grades, or grades of elements of interest in polymetallic deposits. Most of these variables exhibit dependence relationships often expressed as a linear correlation coefficient. The separate estimation of these variables does not reproduce such relationships and may lead to a misleading appraisal of the deposit. Moreover in the mining industry the variables ratio are important for prediction of ore performance process and many place are calculated from ratio estimated variables. These are the reason why the design and application of multivariate estimation methods that consider the dependence relationships between variables is an interesting issue for the mining industry.

This paper presents a geostatistical approach to jointly estimate two ramdon function (eg.: total and soluble copper grades variables), based on two existing methods: maximum/minimum autocorrelation factors and multigaussian kriging.

2 Current approches for estimating many RFs.

2.1 Independent estimation

This approach, still widely used in the mining industry, consists in estimating total and soluble copper grades separately, using inverse distance weighting or ordinary kriging. The estimated soluble copper grade is then corrected, in order to be less than or equal to the estimated total copper grade.

2.2 Cokriging

Cokriging is the multivariate extension of kriging[10]. It relies on a coregionalisation model, consisting of a set of direct and cross variograms that represent the spatial correlation structure of the RFs. This method is interesting when one or several variables are under-sampled (heterotopic sampling) [10] and present linear relationships, but is not quite usual because of the extra time involved in fitting the coregionalisation model. Even in the isotopic case, this approch does not ensure an adequated reproduction linear relationships for estimated random function. Also cokriging does not necessarily reproduce the inequality constraint such as between total and soluble copper grades.

3 Current approches for correlated RFs simulation

3.1 Gaussian co-simulation using a linear model of coregionalisation

The procedure can be summarised as follows:

1. Independently transform each RFs to a Gaussian RFsriable.
2. Calculate the experimental direct and cross variograms of the Gaussian variables, then fit a linear model of coregionalisation (multivariate nested structure model) [10].
3. Co-simulate the Gaussian RFs, using sequential or turning bands simulation, conditionally to the available Gaussian data. Cokriging is the back-end core for conditioning [2].
4. Back-transform the simulated Gaussian variables into raw RFs.

As for cokriging, this approach does not ensure the reproduction of the inequality constraint between the simulated total and soluble copper grades, so that a post-processing step is necessary in order to correct for possible inconsistencies.

3.2 Orthogonalisation Approaches

These methods aim to transform the RFs into spatially uncorrelated factors, to independently simulate the factors (typically via a Gaussian simulation algorithm), and to finally back- transform the simulated factors into grades. The differences between methods rely on how factorisation is performed.

3.2.1 Principal Component Analysis (PCA)

PCA linearly transforms a set of cross-correlated variables into factors that are uncorrelated at collocated locations (i.e., at a zero lag separation vector). The transformation is performed by calculating the eigenvectors and eigenvalues of the correlation matrix of the variables. It is important to notice that the absence of correlation at lag zero does not ensure that the factors are spatially uncorrelated (i.e., uncorrelated for any lag separation vector) [10].

The procedure to co-simulate total and soluble copper grades using PCA factorisation is the following[7]:

1. Transform the original RFs into factors uncorrelated, using PCA.
2. Transform each factor into a Gaussian variable.
3. Calculate the experimental variogram and fit a variogram model for each Gaussian variable.

4. Independently simulate the Gaussian variables, conditionally to the data on these variables.
5. Back-transform the simulated Gaussian variables into factors.
6. Back-transform the factors into the original grade variables.
7. Post-process the realisations in order to correct for inconsistencies (soluble copper grade must be less than or equal to total copper grade).

3.2.2 Min/max autocorrelation factors (MAF)

This approach allows creating a set of factors that are uncorrelated at two lag separation vectors: the zero vector and a vector that can be freely chosen by the user [3, 8]. As in PCA, each original variable is a linear combination of the factors. The workflow for joint simulation using MAF is [5]:

1. Transform the original RFs into factors uncorrelated a certain lag, using MAF.
2. Transform each factor into a Gaussian variable.
3. Calculate the experimental variogram and fit a variogram model for each Gaussian variable.
4. Independently simulate the Gaussian variables, conditionally to the data on these variables.
5. Back-transform the simulated Gaussian variables into factors.
6. Back-transform the factors into the original grade variables.
7. Post-process the realisations in order to correct for inconsistencies (soluble copper grade must be less than or equal to total copper grade).

3.2.3 Stepwise conditional simulation

Stepwise conditional transformation converts a set of cross-correlated variables into a set of Gaussian variables that are uncorrelated at lag zero, by using a hierarchical quantile-quantile transformation[6]. The transformation can be summarised as follows, in the case of two variables:

1. Define a hierarchy for the variables to be transformed, e.g. total copper grade first, then soluble copper grade.
2. Transform the first variable into a Gaussian variable.
3. Transform the second variable into a Gaussian variable, conditionally to classes or intervals of the first transformed variable. The two Gaussian variables so obtained are uncorrelated for the zero lag separation vector.

The workflow for joint simulation is:

1. Perform stepwise conditional transformation to the total and soluble copper grades.
2. Independently simulate the Gaussian variables.

3. Back-transform the simulated Gaussian variables as per the quantile-quantile transformation used in the first step.

As a result of this procedure, a set of conditional realisations of the grade variables are generated. Since the quantile-quantile transformation is not necessarily linear, this approach allows reproducing complex non-linear relationships between the variables, such as inequality constraints.

4 Proposed Approach

The idea of jointly estimating variables by considering their relationship at a zero lag separation vector has been explored in [1], where the stepwise approach is used to transform soluble copper grade (sCu) into a Gaussian variable uncorrelated with total copper grade (tCu). The Gaussian variable associated with sCu is then estimated and back-transformed to sCu. However, this approach may introduce biases due the non-linear nature of the Gaussian transformations of sCu and tCu and to the smoothing effect of estimation. Another approach that also uses stepwise transformation is presented in [4], where the Gaussian variables are estimated with multigaussian kriging, as it is done in this paper.

The proposed approach is based on a combination of min/max autocorrelation factors and multigaussian kriging, therefore a review of the latter technique is presented first.

4.1 Multigaussian Kriging (MGK)

Multigaussian kriging is a method to calculate the conditional distribution of a Gaussian variable. It has been used to calculate the risk of exceeding or of falling short of a given threshold for a continuous (not necessarily Gaussian) variable. It relies on the multivariate Gaussian distribution and on the orthogonality property of simple kriging, which states that the simple kriging estimator is uncorrelated with any linear combination of the data[2].

Under the multigaussian hypothesis, the distribution of a Gaussian variable is fully defined by its first- and second-order moments: mean value and covariance function. Besides, at a given location, the distribution conditional to a set of neighbouring data is still Gaussian, with mean equal to the simple kriging estimate and variance equal to the simple kriging variance. The workflow to get the conditional distribution of a continuous variable is described below:

1. Transform the variable into a Gaussian variable. Store the transformation table.
2. Using the available Gaussian data, perform simple kriging of the Gaussian variable. At each target location, the conditional Gaussian distribution is fully defined by the simple kriging estimate and simple kriging variance.

3. Perform numerical integration at each target location:
   a. Sample the conditional Gaussian distribution using Monte Carlo simulation.
   b. The distribution of back-transformed values is an approximation to the distribution of the original variable conditional to the available data. From this distribution, several measures can be derived, such as the expected value (mean of the distribution), conditional variance (variance of the distribution), probability to exceed or to fall short of a given threshold, or confidence intervals.
   c. Back-transform every simulated Gaussian value according to the transformation table.

4.2 Min/Max Autocorrelation Factors and Multigaussian Kriging Estimation (MAF-MGK)

The proposed approach is similar to joint simulation using min-max autocorrelation factors, with the difference that the Gaussian conditional simulation step is replaced by multigaussian kriging in order to obtain estimated values of total and soluble copper grades. The steps of the proposed approach are:

1. Transform tCu and sCu into Gaussian variables YtCu and YsCu.
2. Apply MAF transformation to YtCu and YsCu in order to obtain two uncorrelated factors F1 and F2.
3. Calculate the experimental variograms of factors F1 and F2. Separately fit variogram models.
4. Separately perform multigaussian kriging to both factors. At each target location, one obtains the conditional distributions of F1 and F2.
5. Separately sample the conditional distributions using Monte Carlo simulation. One gets a set of simulated pairs of factors (f1(i), f2(i)): i = 1... n.
6. Back-transform each pair of factors into a pair of Gaussian variables (ytCu(i), ysCu(i)): i = 1... n, using the inverse MAF transformation.
7. Back-transform each pair of Gaussian variables into a pair of total and soluble copper grades (tCu(i), sCu(i)): i = 1... n.
8. From the distributions of simulated total and soluble copper grades, compute the mean values as the estimates at the target locations. Other measures such as variances or exceedance probabilities can be computed, as mentioned above for multigaussian kriging.

The proposed methodology should be applied at a point support and cannot handle heterotopic samplings, as both grade variables are required for MAF transformation. This approach has consuming CPU time less than simulations approaches.

This method is simple and easy to perform using Isatis software where MAF transformation and multigaussian kriging are already available.

Fig. 1 Plan view of sample locations associated with the exotic mineralisation at MMH

5 Application to a mining data set

5.1 Presentation of the data

The proposed approach is applied to the black oxides unit associated with the exotic mineralisation of the Mina Ministro Hales (MMH) deposit, owned by Codelco Chile. A set of 1289 diamond drill hole samples, located in a volume of 0.7 km (east) , 1.5 km (north), 0.18 k m (depth), are available, with information of total and soluble copper grades, see Fig. **??**

The steps associated with the proposed approach are presented below.

5.1.1 Gaussian transformation of the data

The tCu and sCu distributions are separately transformed into standard Gaussian distributions.

5.1.2 Orthogonalisation with MAF

Given the Gaussian variables of the previous step, a MAF transformation is used to get two factors F1 and F2. A lag distance of 50 m is considered to construct factors that are uncorrelated at this distance, as well as at the zero distance. The MA

F transformation matrix, which allows converting the Gaussian variables into factors, is the following:

$$A_{MAF} = \begin{pmatrix} -1.0734 & 1.4678 \\ 1.7037 & -0.6357 \end{pmatrix}$$

5.1.3 Multigaussian kriging of factors F1 and F2

By construction, the factors are Gaussian and uncorrelated at two lag distances, so they are hypothetically independent. Separate estimations by multigaussian kriging are performed in order to get their conditional distributions. Table 1 presents the variogram models used for F1 and F2.

Table 1 Variogram models of the factors

		Range (m)			
Variable	Structure	Dir. 1: 60/0	Dir. 2: 150/0	Dir. 3: 0/-90	Sill contribution
F1	nugget				0.39
	spherical	150	1000	1000	0.20
	spherical	180	10000	1000	0.12
	exponentiall	300	10000	12	0.29
F2	nugget				0.23
	spherical	65	90	12	0.38
	spherical	65	260	38	0.40

5.1.4 Modelling the conditional distributions of tCu and sCu

The conditional distributions of total and soluble copper grades are numerically derived from that of the factors, by successively applying the MAF back-transformation matrix and the inverse Gaussian transformation:

$$A_{MAF}{}^{-}1 = \begin{pmatrix} -0.3496 & 0.8072 \\ 0.9369 & 0.5903 \end{pmatrix}$$

5.1.5 Post-processing

Given the conditional distributions of tCu and sCu, it is possible to calculate measures such as the expected values of these distributions (used as local estimates for tCu and sCu), their variances (used as measures of uncertainty), or joint probabilities for tCu and sCu, among others.

5.2 Results and Discussion

To assess the performance of the proposed approach, local estimations of total and soluble copper grades using ordinary kriging and cokriging are presented. The comparison is performed over a grid restricted to the geological unit under consideration. The size of the grid mesh is small enough to consider the results as point-support estimates, and the same search parameters are used for the three approaches.

Figure 2(**??**) presents the scatter diagrams between tCu and sCu for the sample data in the top left corner, the ordinary kriging estimates (top right), the ordinary cokriging estimates (bottom left), and for the proposed approach (bottom right). One notes a large amount of grid nodes where the estimated soluble copper grade is greater than the estimated total copper grade in the traditional approaches, whereas the proposed method presents fewer nodes with such inconsistencies and reproduces

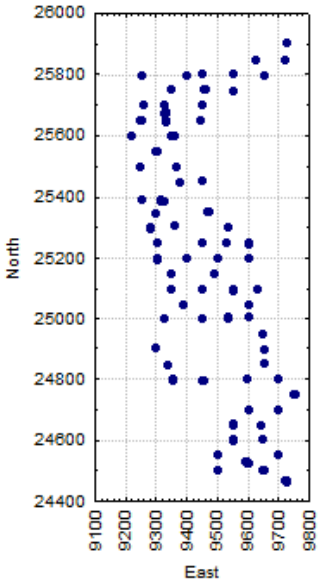

Figure 1 Plan view of sample locations associated with the exotic mineralisation at MMH

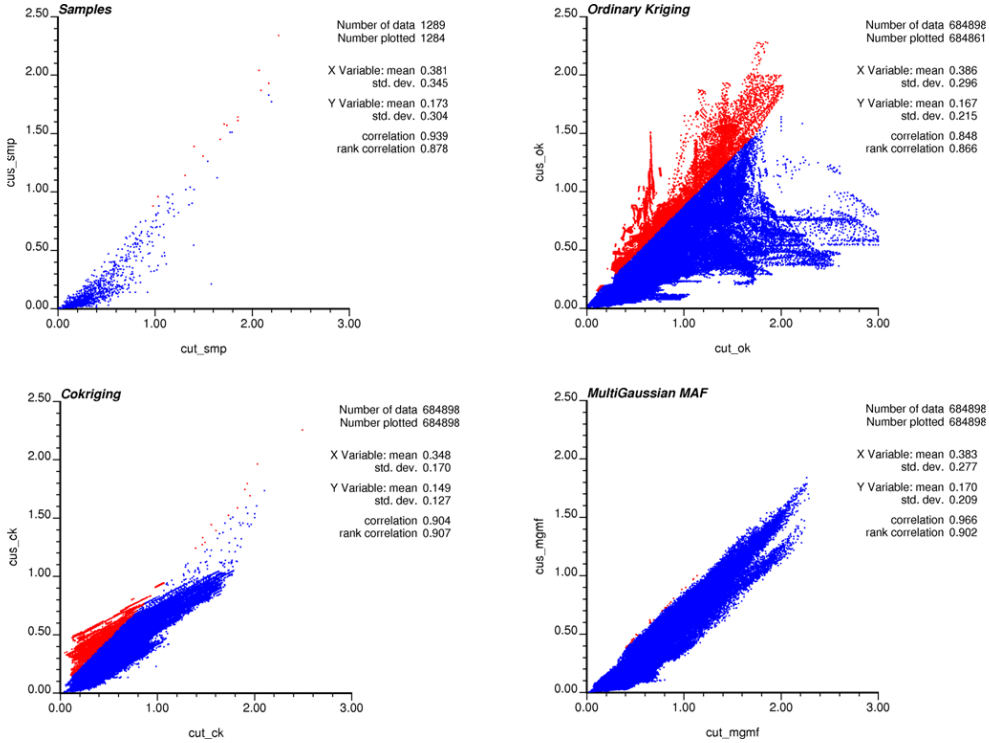

Figure 2 Scatter diagrams between *tCu* (abscissa) and *sCu* (ordinate)
for the sample data and for the estimates using different methods