On the Use of MultiGaussian Kriging for Grade Domaining in Mineral Resource Characterization

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Abstract: In the mining sector, grade domains are often considered, following the modelling of geologic domains, to further control the distribution of grades during resource estimation. This is usually achieved by wireframe modelling on sections displaying grade assays or composites, indicator kriging, and/or via boundary modelling using radial basis functions. This paper proposes an alternative approach to conventional grade domaining, an approach that is based on MultiGaussian kriging. The method consists of estimating grades using MultiGaussian kriging; however, instead of back transforming to obtain a grade estimate at each location, the probability to exceed certain grade thresholds are determined and grade domains are categorized accordingly. This permits uncertainty assessment of grade domains by post-processing for various grade thresholds. An example to a gold deposit is shown, with visual comparisons to grade shells based on radial basis functions that were used to facilitate explicit domain wireframing for resource estimation. Results show the MultiGaussian kriging approach to grade domains can yield comparable shells to existing implicit modelling approaches such as radial basis function. The method and model parameters are data-driven, resulting in an approach that is tractable and repeatable, with the added benefit of quantifying a measure of confidence associated to the resulting grade shells.

Introduction

Geological domaining or modelling is a pre-requisite to resource characterization. Early in the exploration phase of a project, geologists integrate geological logs with drill hole assay data to generate a conceptual geological model. This facilitates geologic understanding and interpretation as more drill data becomes available. Later, when a resource model is initiated, these domains may be used to

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constrain grade estimation/simulation and often satisfy the need for stationary domains.

Geologic domains generally capture regions of lithological units (some more susceptible than others to be mineralized) based on geologic codes, that are often interpreted in the field from one or more geologists. For example, consider an orogenic gold deposit where tens to hundreds of different lithology codes may be used to differentiate between various mafic rocks, ultramafic rocks, sedimentary rocks, felsic dikes and quartz-carbonate veins. The primary objective of these categories is to differentiate core intervals based on one or a combination of factors which may include mineralogy, color, grain size, and/or alteration minerals and metal abundance [12].

This type of data is commonly considered as discrete, that is, a certain lithology is either present or absent at any particular location. This information is then used to construct a three-dimensional (3D) interpretation of the geology to identify regions that are mineralized, within which mineral or metal grades may then be estimated to delineate a mineral resource. This constrains the grade estimation to only those geologic units that may be mineralized.

In the mining sector, it is common to also consider grade domains to further control the distribution of grades during resource estimation. The objective of grade domains is to prevent smearing of high grades into low grade regions and vice versa. The definition of these domains should be based on an understanding of grade continuity and the recognition that the continuity of low grade intervals may differ from that of higher grade intervals [8, 13].

Grade domains may be constructed via a spectrum of approaches, ranging from the more-time consuming sectional method to the fast, semi-automatic boundary or volume function modelling methods. The difference between these two extremes has also been termed explicit versus implicit modelling, respectively [3, 4, 5]. Of course, with today's technology, the former approach is no longer truly 'manual' but commonly involves the use of some commercial general mine planning (GMP) package to generate a series of sections, upon which polylines are digitized to delineate the grade domain. These digitized polylines are then linked from section to section, and a 3D triangulated surface can then be generated. This process can still take weeks, but allows the geologist to have the most control on interpretation. The other end of the spectrum involves the use of a fast boundary modelling approach that is based on the use of radial basis functions (RBFs) to create iso-grade surfaces [2, 3, 4, 5]. With commercial software such as Leapfrog, it can take as little as a few hours to create grade shells.

In practice, it is quite common that these two extreme approaches are used in series to generate reasonable grade domains. The boundary modelling method is first applied to quickly generate grade shells, which are then imported into the manual wireframing approach and used, in conjunction with the projected drill hole data, to guide the digitization of grade domains.

Stegman (2001) used case studies from the Australian gold deposits to demonstrate the importance of grade domains, and highlights several practical problems with the definition of these domains [13]. This paper aims to provide a new approach for developing grade domains that offers the possibility of addressing some of the problems identified by Stegman: incorrect directions of grade continuity and domains that are either too broad or too restrictive.

This paper proposes an alternative approach to RBFs for generating grade domains using MultiGaussian kriging [14, 15, 16]. Similar to RBFs, this method is also data-driven, but with the added benefit of explicitly accounting for grade continuity and providing a measure of confidence associated to any particular grade domain. The background and proposed methodology are first described, followed by an application to a gold deposit. Visual comparisons with an RBF approach are provided, and implementation details are discussed.

MultiGaussian Kriging

MultiGaussian (MG) kriging has traditionally been used to estimate grades, with the added benefit of assessing local uncertainty. Verly (1983, 1986) showed the method was particularly useful in the case of estimating local recoverable tonnages of ore and its associated grade [14, 16]. The method also yielded a quantitative assessment of confidence limits in the recoverable quantity and quality.

MG kriging amounts to the estimation of the normal score transform of the variable or element of interest. For instance, estimating gold grade using this approach requires a transformation of gold grades from its skewed distribution to a symmetric, standard Gaussian distribution. Spatial analysis and estimation is then performed on these normal score values. Kriging of the normal scores provides a local estimate and estimation variance, which is particularly useful given that in a MG context, these two parameters fully define the local distribution of uncertainty. Back transformation via a graphical transform can be easily performed at each location to calculate the local mean and variance, as well as any other summary statistic at each location [10, 11].

Proposed Methodology

Following exploratory data analysis of the geologic domains, the following methodology is proposed for further sub-domain analysis within the geologic domains on the basis of grade classes:

1. Decluster the grade distribution.
2. Transform the data to normal scores.
3. Calculate and model directional variograms for the normal scores.

4. Perform simple kriging on the normal scores of the data.
5. Use numerical integration to back transform the normal distribution to original units. For this task, the GSLib-compatible POSTMG program was used [6, 10, 11].
6. For possible grade thresholds (used to separate grade domains):
   a. Use the local distribution of uncertainty in the grade variable, and then determine the corresponding local probability of the grade to be greater than that grade threshold as $P(Z > z_c) = 1 - P(Z \leq z_c)$ where z_c represents the grade threshold (see Figure 1).
7. Generate iso-probability contours to visualize the resulting grade shells. This may be performed using any GMP that permits the creation of iso-shells from a grid of points. If required, clean the iso-shells to remove any small isolated solids with too few triangulated facets.
8. Select grade and probability thresholds by comparing the various iso-grade and iso-probability shells to determine reasonableness and continuity of the resulting domains.

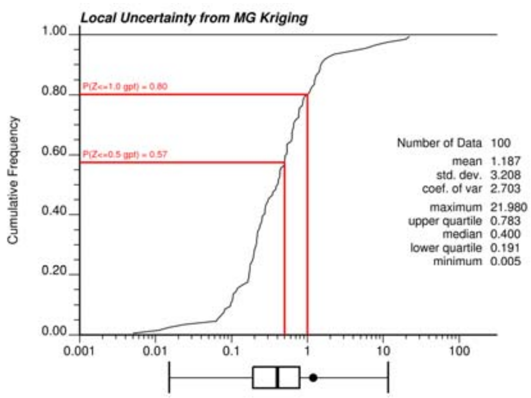

Figure 1 Determination of probability to exceed a grade threshold using the local distribution of grade uncertainty accessible via MG kriging.

The initial five steps in this methodology simply describe the MG kriging workflow. The main contribution of this approach lies in the post-processing of the MG kriging results. Rather than a back transformation to determine the local estimate, we only need to calculate the probability to exceed a grade threshold. These probabilities are then used to contour iso-probability shells. Since the local distributions are fully accessible, post-processing the MG kriged results is straightforward and efficient. There is no need to interpolate the grade values beyond the initial MG kriging model. Any grade threshold can be specified and the associated probabilities calculated.

As this approach requires the creation of 3D contours around the probabilities associated with a particular grade threshold, it is recommended that the results of the MG kriged model should be smooth. Consequently, the generation of this model should be based on a large number of data and a search specification that calls on relatively equal spatial representation of the surrounding data.

Application to a Gold Deposit

Gold assays from the Rainy River gold property located in northern Ontario, Canada were used for the sole purpose of illustrating the proposed methodology. Gold mineralization is described as a hybrid between early gold-rich volcanogenic sulphide mineralization and late shear-hosted mesothermal gold mineralization [1].

The available database for this exercise considers only those drill hole intersects within the main portion of the deposit available up to the end of 2010. A total of 569 boreholes with 68,457 assay intervals are considered for this example. Over 95% of the gold assays are sampled at 1.5 metres or less; these are used directly for this case study. Further, no grade capping or cutting was applied as our primary interest lies not with the estimated grades, but with assessing whether we can find smooth contiguous regions that can be used to further constrain grade estimation. The estimation is also performed in Gaussian units, and as a result, any skewness in the distribution is mitigated through the pre- and post-transformation.

All assays were transformed to Gaussian units, and normal scores variograms were calculated and modelled. The grade continuity is strongly anisotropic with the major continuity direction aligned in the down-dip direction of approximately 250 degrees azimuth and dipping 40 degrees (see Figure 2). The experimental variogram and fitted model are shown in Figure 3 and given below:

$$\gamma(\mathbf{h}) = 0.1 + 0.42\,Exp_{\substack{\text{ahmax=10}\\ \text{ahmin=35}\\ \text{ahvert=11}}}(\mathbf{h}) + 0.13\,Sph_{\substack{\text{ahmax=90}\\ \text{ahmin=35}\\ \text{ahvert=68}}}(\mathbf{h}) + 0.35\,Sph_{\substack{\text{ahmax=650}\\ \text{ahmin=450}\\ \text{ahvert=68}}}(\mathbf{h})$$

where ahmax corresponds to the down-dip direction, ahmin corresponds to the along-strike direction and ahvert corresponds to the direction that is perpendicular to the plane-of-the-vein.

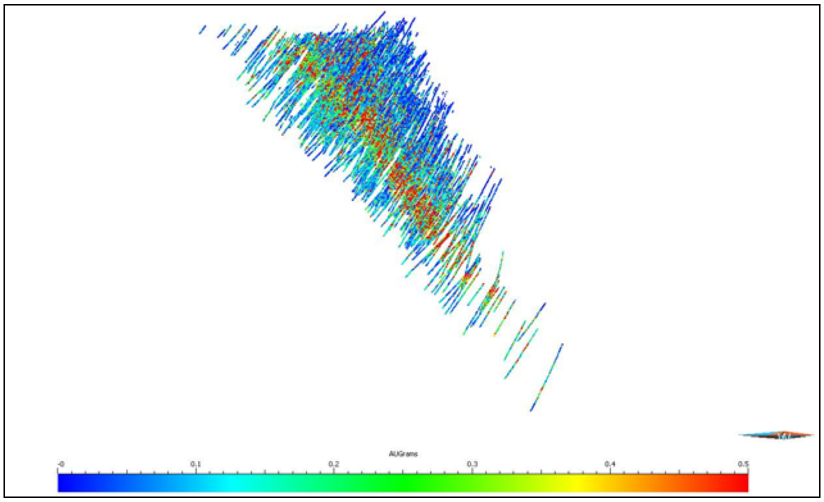

Figure 2 View of the gold assay values looking east.

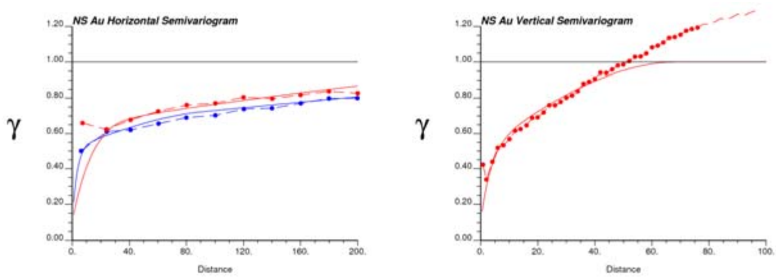

Figure 3 Normal scores variogram for gold in the plane of vein directions (left) and parallel to vein thickness (right). Blue line on the left corresponds to the dip direction, red line corresponds to the along-strike direction.

MG kriging was then performed using the normal scores data and its corresponding normal scores variogram. A grid of 10 metres x10 metres x10 metres was considered for this estimation, requiring a total of 6.75 million nodes to be estimated in order to cover the extents of the data plus at least 500 metres on either side of the data extents. To ensure a relatively smooth model of probabilities will be obtained, the maximum number of data specified was 64, with an octant search using a maximum of 8 data per octant. These parameters were not optimized, though some sensitivities related to the number of data and grid resolution were performed. Further, simple kriging was specified as the estimation approach.

The MG estimation results are then post-processed at each location to determine the probability to exceed specified grade thresholds. For this project, three grade shells were previously selected based on RBF results, corresponding to cut-offs of 0.3 grams per tonne (gpt), 0.5 gpt and 0.9 gpt gold. These same thresholds are chosen for comparative purposes. The result of this post-processing step is a 3D model of probabilities to exceed the specified grade threshold, with the same dimensions as the MG kriged model.

For the three chosen grade thresholds, the 3D model of probabilities was then imported into gOcad to create iso-probability shells. Other GMPs likely offer similar functionality, but the nice feature found within gOcad is the efficient manner in which these solids can then be cleaned to filter out solids with too few triangulated facets.

Figure 4 and Figure 5 show a comparison of the iso-probability shells for a grade threshold of 0.5 gpt gold in a 3D and a 2D section, respectively. Three probability thresholds corresponding to the 30%, 40% and 50% probability to exceed 0.5 gpt gold are shown. As expected, for the same grade threshold, there is a natural nesting of the iso-probability shells obtained. Increasing the probability threshold, and thereby requiring greater confidence in the predictability of the grade domain, results in smaller, more restricted grade shells. Conversely, relaxing the probability threshold yields broader grade domains.

A similar nesting can be observed for different grade thresholds using the same iso-probability value. While this makes sense, it is not necessarily a good idea to choose a constant iso-probability value to apply to all relevant grade thresholds. This is discussed further in the next section.

Another interesting observation is that there appears to be a probability value for which the resulting domain is too large, and above which, more reasonably contiguous zones appear. For the 0.5 gpt gold threshold shown in Figure 4, this threshold probability value lies somewhere between 30% and 40% probability.

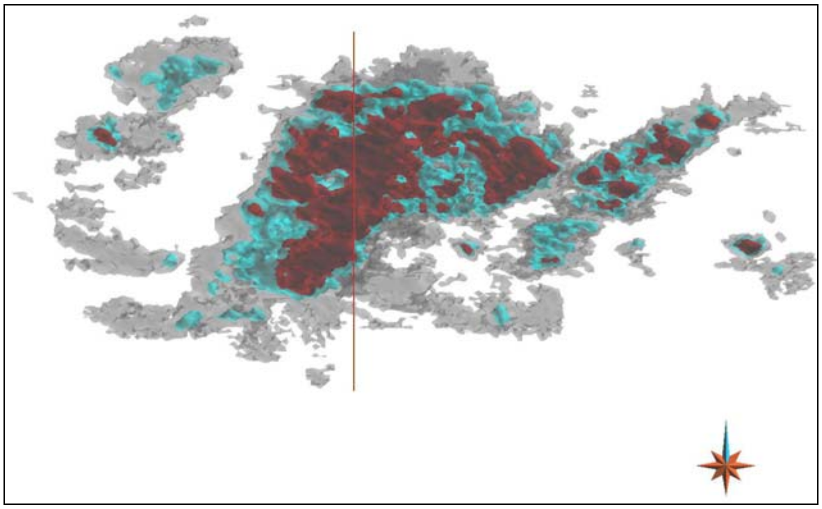

Figure 4 Comparison of iso-probability shells for 0.50 gpt grade shell: probability to exceed grade 0.5 gpt of 30% or higher (grey), 40% or higher (cyan), and 50% or higher (red). Red vertical line corresponds to location of 2D section shown in Figure 5.

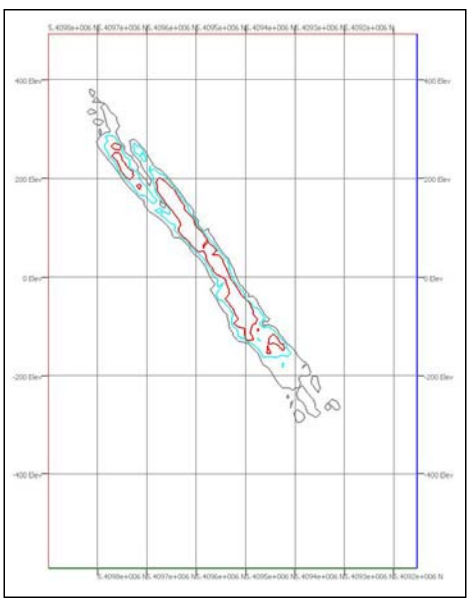

Figure 5 Section at 425405 Easting, looking east, illustrating the nesting of iso-probability shells for 0.50 gpt grade threshold: probability to exceed grade 0.5 gpt of 30% or higher (grey), 40% or higher (cyan), and 50% or higher (red).

The RBF grade shells were superimposed on the MG grade domain shells for a visual comparison of the two different approaches. Note that the RBF shells were generated by geologists independent of the authors, and these were used to facilitate and expedite the manual wireframing (or explicit modelling) process for a previous resource model. Figure 6 and Figure 7 show a comparison of the two approaches for a grade threshold of 0.50 gpt gold; an iso-probability value of 40% appears to give similar domains, in 3D and 2D sectional view, respectively. For the 0.30 and 0.90 gpt gold thresholds, we can find an iso-probability value that yields similar domains to the RBF shells (50% and 30%, respectively). In general, the MG domaining approach can generate comparable grade shells based solely on the choice of the probability threshold.

It is interesting to point out that this method provides a quantitative measure of confidence in the domain volume that was modelled using RBF. A few geologists who have seen this example expressed some dismay at the corresponding iso-probability values chosen for the same grade thresholds; many were surprised at the relatively low probabilities associated to the RBF shells.

Discussion

The previous case study showed that the MG kriging approach can be useful in generating comparable grade shells, which can later be used as a guide during digitization of grade domains. This should expedite the geological wireframing process. The results likely should not be used directly as grade shells, unless the resulting iso-probability shells are sufficiently smooth and/or cleaned to generate reasonable, contiguous regions for resource estimation.

The degree of smoothing required for this type of modelling implies that the conventional estimation scheme common in the mining sector is inadequate. The objective of this estimation is not to provide local or even global accuracy, but to yield large, continuous zones that are relevant for resource modelling. As such, a large number of data should be chosen – how large depends on the amount of time available for this type of modelling. In this case study, more than 50,000 assay intervals were available to construct a model of over 6 million cells. With a maximum of 64 data allowable, the estimation took less than one hour to run. It is suggested that at least 50-100 data should be considered for estimation in this context while still being practical from a run-time perspective.

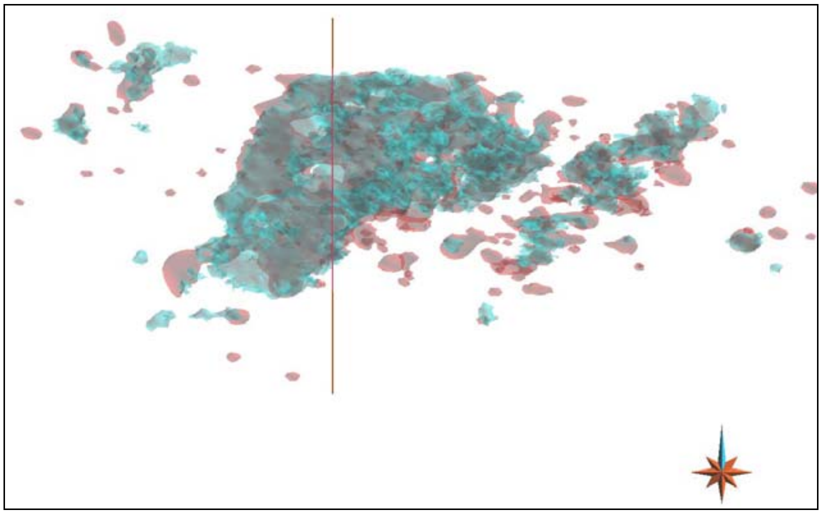

Figure 6 Comparison of 0.50 gpt gold grade shell from RBF (red) and a similar shell using MG domaining for an iso-probability of 40% (cyan). Red vertical line corresponds to location of 2D section shown in Figure 7.

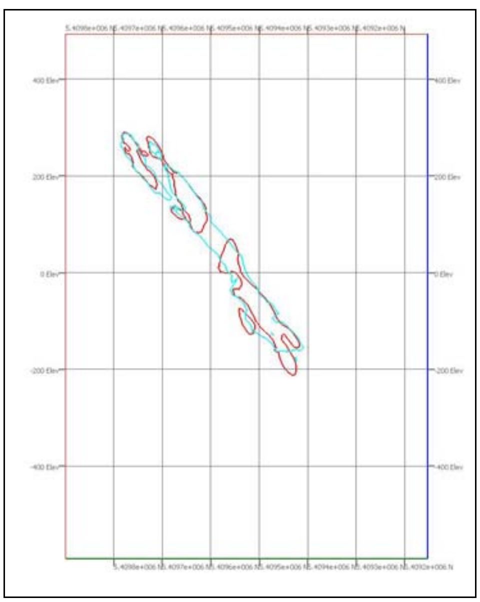

Figure 7 Section at 425405 Easting, looking east, comparing the 0.5 gpt gold grade shell from RBF (red) with the MG domain shell for an iso-probability of 40% (cyan).

Post-processing takes no more than a few minutes, importing into a GMP package and generating iso-shells may take as little as an hour to visualize and compare different grade thresholds and iso-probability shells. The time cost of this approach is clearly upfront. MG kriging requires specification of a variogram – this should correspond to the normal scores of the data. While some may find this a prohibitive, time-consuming task, it should provide greater fidelity to the data if indeed the spatial correlation imposed on the model were actually analyzed and appropriately fitted rather than chosen arbitrarily, as often seems the case in current practice with RBFs. Besides, the approach calls for the normal scores variogram which is often easier to infer than the variogram of the original data, and may provide additional insight into correlation structures relevant down the line when the task of grade estimation is at hand.

The case study also confirmed the expectation that there should be a natural nesting of grade shells. Higher grade shells are nested within medium grade shells, which are in turn encompassed by lower grade shells. This follows intuitively if we consider how the probability to exceed a threshold is determined at each location (Figure 1) and that the shells then are generated as 3D contours at different probability thresholds.

Of course, the choice of a reasonable probability threshold also has an impact on the degree of nesting of grade shells. With higher grade thresholds, we expect that an acceptable probability threshold will be lower than if we considered a lower grade threshold. Consider applying a very high grade threshold. The probability to exceed this cut-off grade would likely be low in most of the field, with a few isolated zones of reasonably high probability centred on the high grade data. Since the purpose of this exercise is to generate domains, which inherently implies some grouping of locations that is continuous and meaningful for estimation, then this naturally entails that we should consider a grade threshold that encompasses a reasonable grouping of original data *and* that we relax the probability to exceed this threshold. This is a fine line to walk since too relaxed a probability criterion yields too large a domain and no longer serves the purpose of constraining grade estimation and too strict a criterion will produce very restrictive domains that may be discontinuous. These are problems noted in [13]. Even though the method presented here does not provide a universal solution, it does provide a framework within which the user can choose where they want to be on the spectrum from too restrictive to too relaxed. Ultimately, the choice of an appropriate probability threshold is subjective. We recommend that for each grade threshold considered, different iso-probability shells should be visualized to determine whether the volumes are sufficiently continuous and contiguous to serve as estimation domains.

This alternative approach to domaining can be applicable to discrete geologic data if a clear physical ordering of the lithologies applies to that particular project setting. This is the same limitation of truncated Gaussian simulation for geological modelling since both approaches rely on a continuous multiGaussian estimation/simulation method, followed by a partitioning of the Gaussian random function to assign a lithology to each location [7, 9]. Consequently, the nesting observed in the grade shells should translate to a nesting of lithological categories

Conclusions

The multiGaussian kriging approach to constructing initial grade domains provides a simple alternative to current indicator kriging and/or RBF approaches. The method relies on geostatistical analysis of the grades, followed by a grade estimation approach that inherently allows access to the local distribution of uncertainty. Unlike other kriging algorithms, the use of MG kriging requires the least amount of inference while still providing a wealth of information.

This approach fills a gap between the more time-consuming, yet rigorous explicit manual wireframe modelling method, and the fast, automatic implicit boundary modelling techniques. It is practical for the construction of grade domains, but is limited in application to geologic domains due to the inherent nesting of resultant domains.

The benefits of this proposed method include: (1)sensitivity assessment on appropriate grade threshold(s) can be performed since this decision is made post-estimation; (2) inference of normal scores variogram is often easier than original units and/or indicators, and requires little additional effort; (3) extension to multiple grade domains, such as low, medium and high grade is not only easy but naturally progressive using this approach; (4) the method is computationally more efficient than manual wireframing, and; (5) the method is traceable, repeatable and defensible.

Acknowledgments

The authors wish to thank Rainy River Resources for permitting the use of their data for the application of this methodology.

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