Cokriging Partial Grades - Application to Block Modeling of Copper Deposits

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Abstract This work concerns mineral deposits made of geological bodies such as breccias or lenses that contain several categories of grades with different characteristics in terms of distribution and variogram. When production blocks contain few such bodies, estimating block grades by ordinary kriging may produce unrealistic spatial continuity. We propose a method based on the indicators of objects (units or facies) together with their products with the grade. This is illustrated by an application to a porphyry copper deposit.

Introduction

We try to answer this question: Given samples informed by a categorical variable and a grade, what is the best way to estimate the average grade at the scale of production blocks? We propose to split the grade in a sum of "partial grades", which leads to an isotopic cokriging system based on the indicators of the units and their products with the grade. In an application to a porphyry copper deposit, we show how we can characterize the geometry of the units and we build the cokriging system. The resulting block model is compared to usual kriging.

This expanded abstract summarizes an application to a porphyry copper deposit located in northern Chile and developed in [1]. The oral presentation is based on a second, unpublished, case study.

Basic functions used in the sequel are indicators. The probabilistic interpretation of a simple or cross variogram and on the ratio of a cross variogram by a simple one is presented by Rivoirard [2]. A general overview of these tools is given by Chilès and Delfiner [3].

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Formalization

We consider an ore body where the ore is classified in n subsets or units (ore types or facies) and denote by $1_i(x)$ the indicator of unit i:

$$1_i(x)=1 \text{ if } x \in \text{unit i, 0 if } x \notin \text{ unit i} \quad (1)$$

We consider that we have as many grade variables as units: The grade $Z(x)$ at point x is decomposed in "partial grades"

$$Z_i(x)=1_i(x)Z(x) \quad (2)$$

At point x, these partial grades are equal to zero, except one which is equal to $Z(x)$. Their sum is thus equal to $Z(x)$.

Let us now consider a block V. The partial grades of block V are defined by

$$Z_i(V)=\frac{1}{V}\int_V Z_i(x)dx \quad (3)$$

The grade Z(V) of block V is the sum of the partial grades $Z_i(V)$.
When the partial grades do not have the same spatial structure, it is sensible to estimate Z through the Z_i's rather than directly. We are in an isotopic situation (the variables are equally sampled) and the cokriging of the sum equals the sum of cokrigings

$$Z(V)^{CK}=\sum_{i=1}^{n} Z_i(V)^{CK} \quad (5)$$

For each grade Z_i, we estimate by cokriging its average over V and we add the estimations.

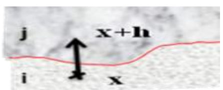
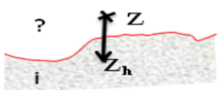
We have 2n variables to consider for building the cokriging system

- n unit indicators $1_i(x)$ (introduced for their major influence),
- n partial grades $Z_i(x)$ (variables of interest).

Tools

Let us denote by $\gamma_i(h)$ the variogram of the indicator $1_i(x)$, by $\gamma_{ij}(h)$ the cross variogram of the indicators $1_i(x)$ and $1_j(x)$, and by $\gamma_{iZi}(h)$ the cross variogram of the indicator $1_i(x)$ and the corresponding partial grade $Z_i(x)$. Our main tools are the ratios of cross variograms by an indicator variogram. Table 1 shows their probabilistic interpretation.

Table 1 Indicator and grade variograms (denoted by Greek letter γ) and their interpretation

Calculation	Interpretation	Conceptual illustration
$\left\lvert \frac{\gamma_{ij}(h)}{\gamma_i(h)} \right\rvert$	p(x+h ∈ j/x ∈ i, x+h ∉ i) Probability to reach j while leaving i	
$\frac{\gamma_{iZ_i}(h)}{\gamma_i(h)}$	E[Z(x+h)/x+h ∈ i, x ∉ i] Average grade when entering in i	

Application

The volume of the studied domain is approximately 400x1500x400 m^3 and it contains more than 54000 samples of 1.5 meter length, all informed in copper grade and coded in 4 units. (Table 2).

Table 2 Main characteristics of grades

	Abbr.	Color	Proportion %	Mean grade	Std dev.	Min	Max
All units			100	0.78	1.54	0	33
Waste	W		31.2	0.06	0.17	0	8.1
Low grade	C1		27.5	0.31	0.36	0	7.6
High Grade	C5		31.7	1.16	0.90	0	23.1
Breccias	Bx		9.6	4.27	3.48	0	33

Average grades present important differences between units. Even poor units present high grades (8% of copper for waste for example).

Table 3 presents the probability, when leaving a given unit, to encounter another unit. It must be read together with the global proportions of Table 2.

Table 3 Contact probabilities

To / From	W	C1	C5	Bx
W		0.8	0.2	0
C1	0.2		0.65	0.15
C5	0.05	0.5		0.45
Bx	0	0.2	0.8	

Main comments are:

- Bx and W are not in contact
- When leaving W, the probability to enter in C1 is 0.8, which is much greater than the global proportion of C1 (less than 0.3). So C1 separates W from C5 and Bx
- Same remark for the probability to encounter C5 when leaving Bx: C5 separates Bx from C1 and W
- When leaving C1, one can encounter Bx with a low probability (0.15), while the probability to encounter C5 (0.65) is more important than the global proportion of C5. Same remark when leaving C5 and encountering C1. This shows that C5 tends to surround Bx.

Such considerations as well as the knowledge of the geologist make it possible to generate a scheme showing the mutual behaviors of the units (Figure 1).

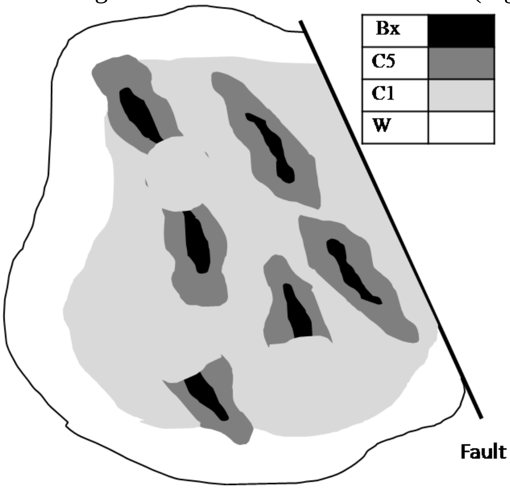


Fig. 1 Schematic representation of the mutual behaviors of the units deduced from the statistical analysis of the contacts

Figure 2 shows ratios of cross variograms by a simple variogram.

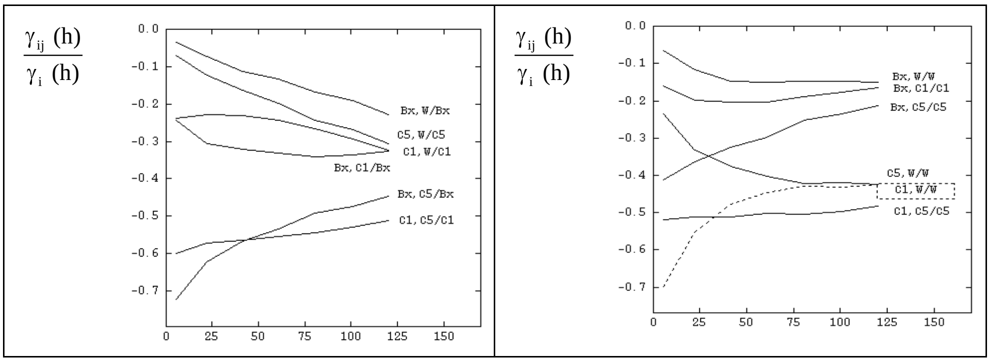


Fig. 2 Ratios of indicator cross variograms by simple a simple variogram.

We notice that, apart one exception, all units present different spatial correlations and therefore must be estimated jointly by cokriging. Exceptions are C1 and C5 where the probabilities do not depend on the distance. The frontier between these two units marks the limit between poor ore (mainly to the west) and rich ore (to the east).

Let us now consider the ratios of cross variograms between the unit indicators and the partial grades by the indicator simple variograms. Figure 3 presents the most representative behaviors.

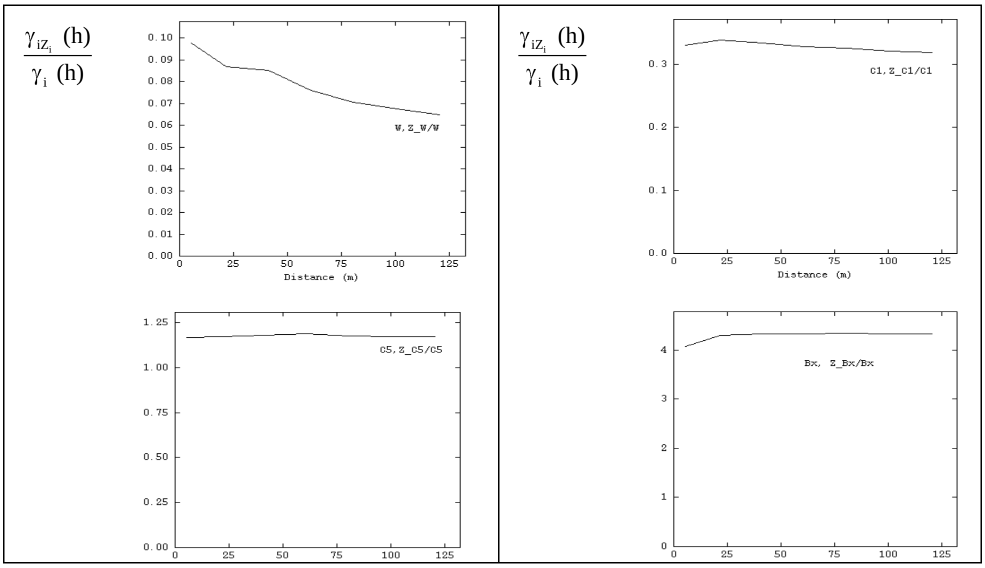


Fig. 3 Ratios of cross variograms between unit indicators and grades by indicator simple variograms

There are border effects for each unit but their magnitude is not important. The upper left variogram of Figure 3 shows that the copper grade in W decreases when moving away from the boundary of W, but the decrease in copper grade is only 0.05% after 150m. The most important gradient is linked to Bx, the average grade increases of 0.40% after 150m (bottom right variogram of Figure 3). Globally, one can consider that grade variations are smaller within the units than between units.

A cokriging system is built incorporating indicators and partial grades. Results are compared block by block to kriging without distinction of the units.

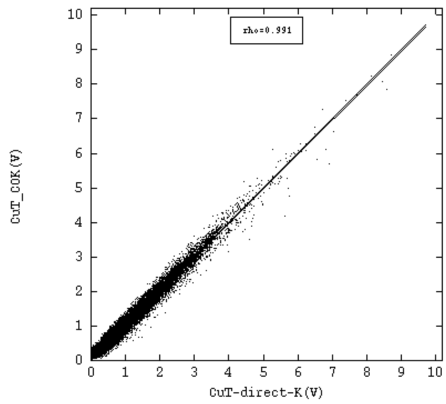


Fig. 4 Scatter diagrams between partial grade cokriging versus usual kriging

The scatter diagram between direct kriging and cokriging shows an important correlation (Figure 4). The standard deviation of the difference between the two estimates is 0.1%. Both estimators give close results. Two reasons can be pointed out:

- The grades follow the sequence W-C1-C5-Bx, which leads to the mutual organization of the facies. When we estimate Z directly using a moving neighborhood, we take into account this sequence naturally because low grades mainly concern W and high grades Bx.
- All the variograms contain more than 50% of nugget effect and this reduces the impact of the estimator choice on the results because an important part of the calculation is just a local average.

Conclusions

The interest of this approach is not located in the resulting estimation, but on the analyses that leads to it.

First this approach enables us to separate the sole geometry of the units (modelled by the indicators) from the behavior of this geometry together with the grade inside the units (modelled by the partial grades). This leads to calculation priorities like for example in the present data set where one must focus on the proportions estimation in each block, and one can then affect to each unit an average grade in the block.

Secondly, present calculations, only based on statistics, could act as a reference for the usual practice which consists in drawing geological objects by hand and intersecting them with the grid to calculate block-by-block proportions. The difference between both approaches quantifies the impact of the geological knowledge on the results. Important differences may indicate a lack of data and an important uncertainty of the resulting block model. The proposed methodology could act as a “handrail” against excess.

Bibliography

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