Inference of 2D and 3D locally varying anisotropy fields for complex geological formations

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Abstract Many geostatistical techniques perform better when anisotropy is considered to be locally varying, but there are few techniques available for inferring the necessary locally varying anisotropy (LVA) field. All geostatistical modeling methodologies assume a form of stationarity; one common assumption is second order stationarity where anisotropy is considered to be globally constant. This assumption is often geologically inappropriate and requires geostatisticians to subdivide the modeling area into stationary domains, thereby increasing the required professional time for modeling and potentially producing disjointed domains that are globally inconsistent. Existing algorithms can be modified to relax the assumption of second order stationarity, for example: multiple point statistics with local pattern reorientation; local kriging search reorientation, and; kriging with LVA. These techniques require an exhaustive model of the local orientation and magnitude of anisotropy, termed an LVA field. In practice there are rarely direct measurements of the LVA field, even local dipmeter data can be unreliable due to the discrepancy between the measurement of small scale anisotropy and the larger block scale anisotropy required for modeling.

The focus of this work is to develop a suite of methodologies for LVA field inference from various 2D or 3D data sources. No single methodology is appropriate for all deposits because local geology and data availability vary. Methodologies presented for inference of 2D LVA fields include (1) manual feature interpolation (2) structure tensor, and (3) polylines. Methodologies for inference of 3D LVA fields are similar but require additional considerations for visualization and checking. Techniques are demonstrated with real data when available, examples include: a porphyry deposit and a uranium role front deposit. Recommendations to aid in technique selection are provided and are largely determined by the nature of the data available and the geometry of the formation under study.

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Introduction

The focus of this work is to incorporate locally varying anisotropy into numerical models. The traditional assumption of second order nonstationarity is inappropriate for complex deposits that display highly nonlinear patterns of continuity; however, many ore reserve estimation techniques rely on kriging to characterize distributions at unsampled locations and assume second order stationarity. Imposing constant anisotropy in models with known locally varying spatial features compromises the accuracy of the numerical model. Rather, increased accuracy is achieved by incorporating locally varying anisotropy. Many techniques incorporate locally varying anisotropy such as (1) using the shortest path distance (Boisvert and Deutsch, 2011) (2) LAK (Stroet and Snepvangers, 2005) (3) MPS with local pattern reorientation and (4) local variogram reorientation (Deutsch and Lewis 1992; Xu 1996). The interested reader is referred to the aforementioned references for incorporation of LVA into numerical modeling; the focus of this work is to model the necessary input LVA field.

The LVA field describes the local orientation and magnitude of the major direction of continuity in a modeling domain. In 3D, the orientation is defined by a strike, dip and plunge angle. The magnitude of anisotropy is described by two ratios: r_1 is the ratio between minor and major axis (typically horizontal); r_2 is the ratio between the vertical and major direction. Only the strike and r_1 is required in 2D. In a single domain a constant magnitude is often an appropriate assumption. Estimating the magnitude of anisotropy is not difficult as the ratios are typically continuous variables that are handled well by traditional techniques. Modeling the orientation of anisotropy is not as obvious as it is an axial directional variable (Mardia and Jupp 2000).

The following sections are focused on inferring the locally varying orientations of anisotropy from different data sources. The methodologies shown deal with either continuous or categorical data of the following types:

I. **Point Source data:** an exhaustive LVA map can be generated from direct measurements, often from a dipmeter tool.

II. **Geological knowledge of the deposit:** a practitioner can interpret the local anisotropy from a detailed geological understanding of the domain. Typically this would entail assigning pseudo data on sections to represent the orientation of the mineralogy.

III. **Exhaustive secondary data:** If an exhaustive secondary variable has been measured that is deemed to have the same spatial characteristics as the variable of interest, the proposed techniques can be used to infer the LVA field of the secondary variable.

1. LVA from point source data

LVA orientations are undirected or axial which means that the continuity of a deposit is indistinguishable in the direction of strike and strike +180°. Thus, axial data are said to 'wrap' at 180° rather than at 360° as with typical directional data. Mardia and Jupp (2000) suggest doubling the data so that the angles wrap at 360^0 and typical vector decomposition of the orientation would be appropriate.

Data that are point source measurements, such as formation dip measurements from a dipmeter, require doubling. Firstly the data is restricted to $0 \le x \le 180^0$ (i.e. a strike of 270^0 is set to 90^0) the recorded values are doubled and transformed into standard circular coordinates:

$$C = R\cos(x) \quad \text{and } S = R\sin(x)$$

where x is the strike angle measured clockwise from North.

The components (C and S) can then be estimated(Figure 1) or simulated with traditional techniques such as kriging, SGS etc. If simulating, it is important to reproduce the correct correlation between C and S and a bivariate technique should be considered. When there is uncertainty in the form of the LVA field, simulation from the point measurement data is recommended. Multiple models, each with a different LVA field, can be generated to incorporate the uncertainty in LVA.

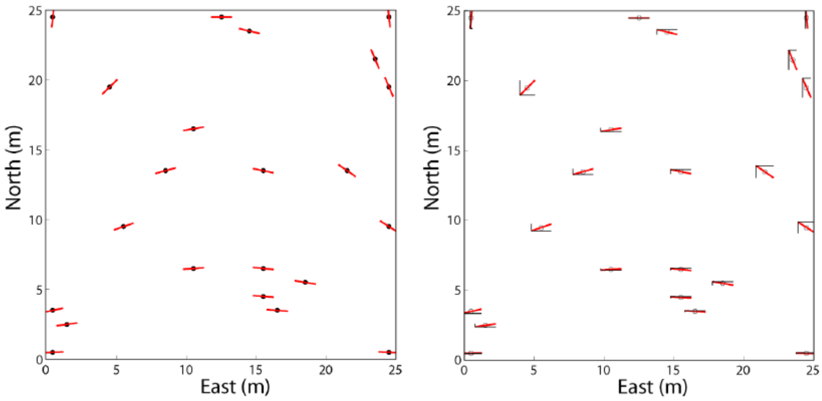

Figure 1: Left- Synthetic dipmeter data. Orientation of lines at each location indicates the local directions. Right- Decomposition of each orientation into X and Y components (Boisvert, 2010).

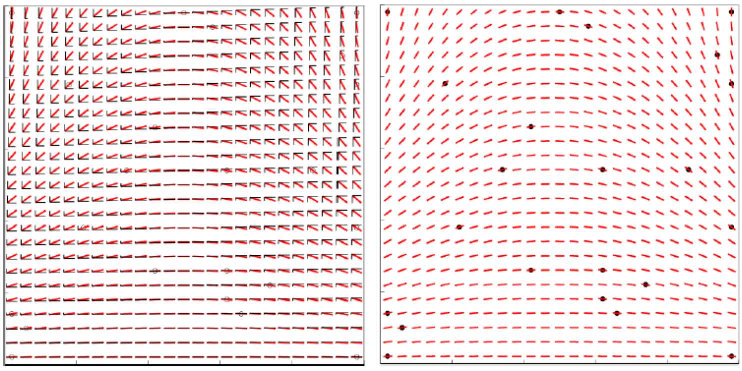

Figure 2: Components are estimated/interpolated at every location and recombined to generate an exhaustive LVA field (Boisvert, 2010).

2. LVA from Geological knowledge

2.1 Manual LVA Inference

This is a straightforward methodology and borrows heavily from expert geological knowledge of the deposit. The LVA field can be generated at various locations based on a visual assessment of the available data. At control points, the practitioner adds pseudo orientation data and an exhaustive field is mapped using kriging and the vector decomposition discussed above. This technique is subjective and should be used if the continuity of the variable of interest follows along known major geological structures.

This method is illustrated in an example below. An expert geologist determines local orientations at discrete points by visual assessment and an exhaustive map is generated with kriging.

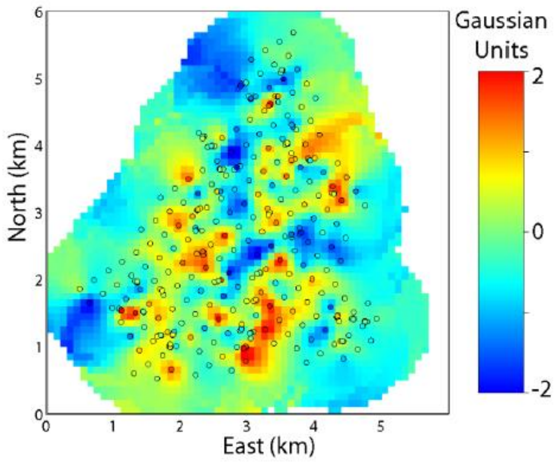

Figure 3: Data is normally transformed and kriged to present an underlying smooth map (Boisvert, 2010).

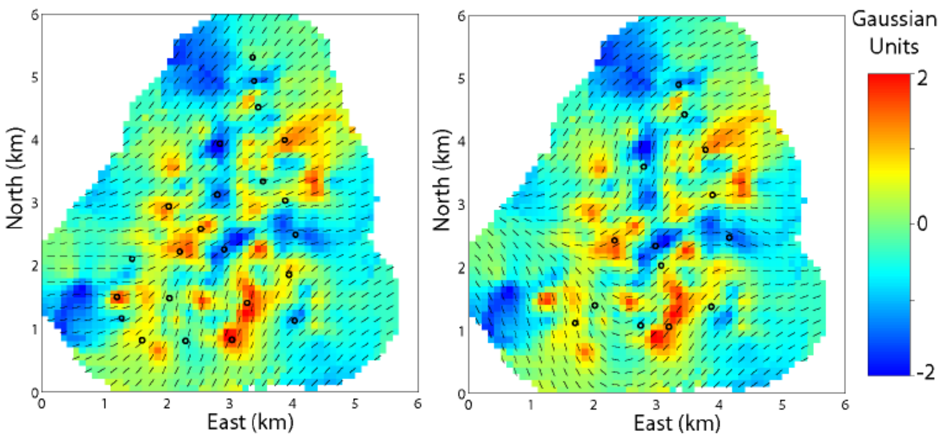

Figure 4: Left – A deterministic assessment of the exhaustive LVA map from expert determined local orientations at 14 points (circles). Right - An alternative interpretation of the LVA field using orientations at 23 points (Boisvert, 2010).

2.2 LVA Inference from Polylines

Many deposits show a very distinct LVA pattern that repeats on section. Consider the well-studied Doris North gold deposit in Nunavut, Canada (Carpenter et al 2003; Fraser 1964; Gebert 1993; Sherlock et al 2002). The deposit has been interpreted to be an anticlinal quartz vein. The data consists of 667 drill holes with a naïve average gold grade of approximately 5g/t and around the solid (Figure 7). The LVA field for this deposit is modeled by manually selecting the limbs and crest of the anticline and fitting a parabola to each section (Boisvert and Lillah, 2012).

Often, using polylines to fit the LVA field is more practical than selecting individual points as discussed previously. Moreover, a polyline provides an exhaustive LVA field (in 2D) and can be extended to 3D when considering a surface.

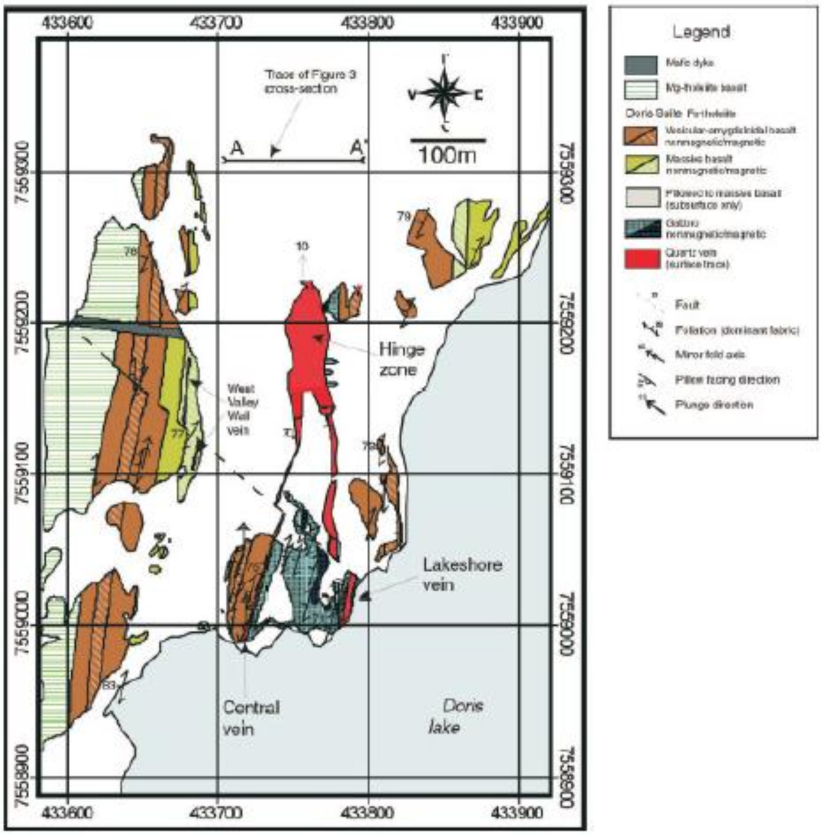

Figure 5: Geometry of the deposit shown as the central vein, hinge zone and lakeshore vein (Carpenter et al 2003).

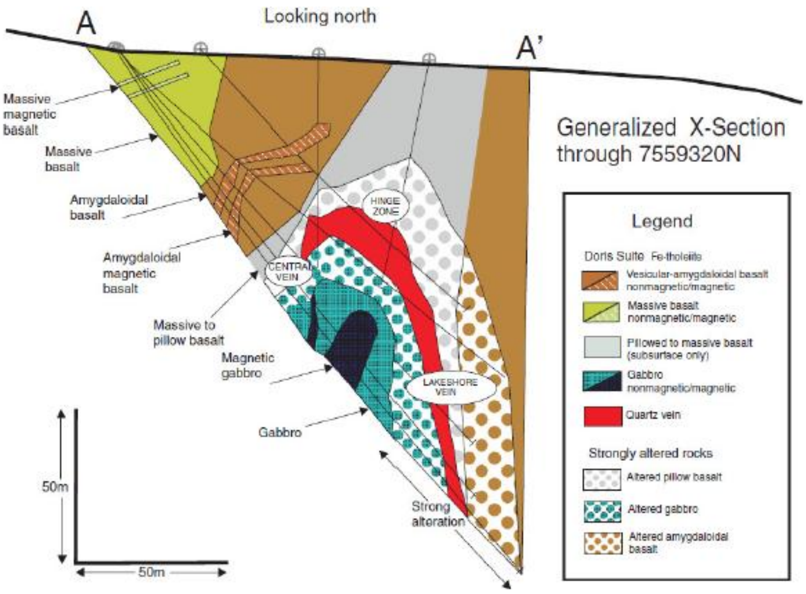

Figure 6: Cross section showing anticlinal geometry (Carpenter et al 2003).

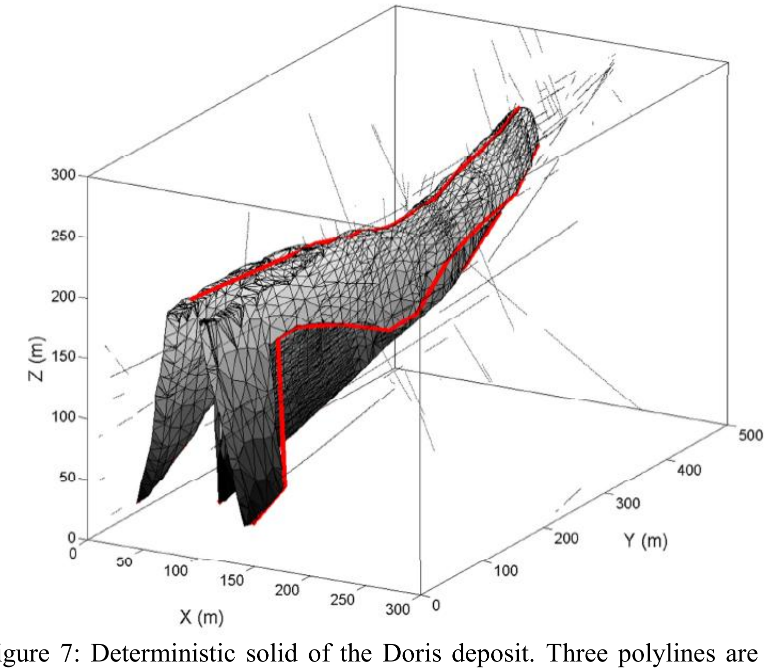

Figure 7: Deterministic solid of the Doris deposit. Three polylines are fit to the limbs and hinge for generating LVA field (Boisvert and Lillah, 2012).

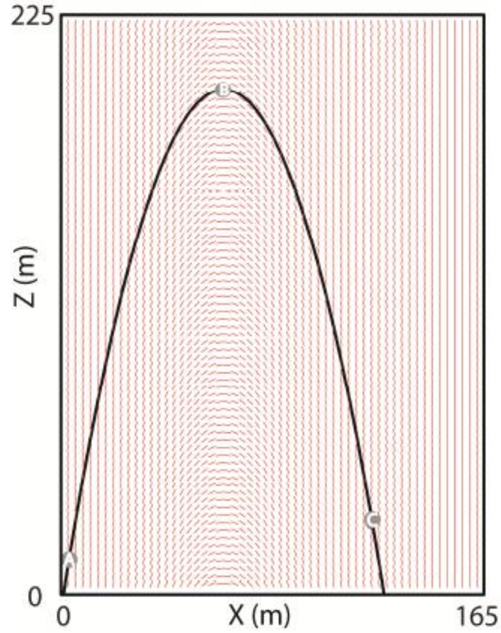

Figure 8: Typical cross section of the LVA field. Interpreted polylines from the figure (grey intersections A, B and C) are fit with parabola to determine the exhaustive LVA. (Boisvert and Lillah, 2012).

2.3 LVA from Surfaces

Generating LVA from surfaces is fairly intuitive. The orientation of the LVA field is taken as the local dip of the surface. Regridding is used to obtain the necessary LVA field on the desired grid. The technique is similar to obtaining LVA fields from polylines.

Often intermediate surfaces can be obtained from exhaustive secondary data such as seismic surveys. Moreover, surfaces are commonly modeled manually from a geological interpretation of the deposit and can be a rich source of data for LVA inference.

3. LVA from exhaustive secondary data

Data from remote sensing techniques provide low-resolution exhaustive secondary variables which can be exploited to obtain the regional continuity of the variable of interest. To use the secondary data for LVA inference, the spatial continuity of the secondary variable must be deemed representative of the spatial continuity of the variable of interest. Moreover, scale is an important issue. Typically the desired LVA scale is at the block model size or larger as it must be representative of the LVA between points separated by at least one block. The scale of seismic data (considered here) may be slightly too large vertically, but is often reasonable for LVA inference.

Several methods can be employed to extract the local orientation from a given exhaustive data set such as (1) the moment of inertia technique (Hassanpour and Deutsch,2008) (2) methodology based on PCA (Feng and Minanfar,2003) and (3) structure tensor proposed in this work.

Local gradients can be calculated for the exhaustive secondary data. The difference in gradients around the neighborhood of a cell is calculated and a Gaussian kernel is used to smooth noise. The resulting orientation is the local orientation at the selected cell.

Components of the tensor matrix:

$$T_{ij}(x_0) = \int h(x_0 - x)\frac{\partial I(x)}{\partial x_i}\frac{\partial I(x)}{\partial x_j}dx \quad . \tag{1}$$

where, $\partial I(x)/\partial x_k$ is the directional derivative of the gradient vector *I(x)* along k^{th} spatial coordinate

Equation (1) is written for continuous data. The equivalent expression for discrete data is taken as,

$$\hat{T}_{ij} = F(R_i R_j) \quad . \tag{2}$$

R_k is a discrete differentiating operator in the k-th direction and the R_iR_j is a simple multiplication of the respective filters. The kernel *F()* here is a Gaussian smoothing operator. The eigenvalue decomposition of the symmetrical *T* matrix gives two components; the greater value corresponds to the direction of maximum signal change for a local neighborhood of pixels.

The following steps are used to determine the LVA field orientation in the spatial domain:

I. Calculate the gradients at each point in the image of the model; R_k is the stored gradient value in the k^{th} coordinate direction
II. Find necessary products of R_i and R_j for every pixel, and apply the Gaussian filter to obtain the components of the tensor matrix T

In 2D the tensor matrix for an image location (i,j) is:

$$\hat{T} = \begin{pmatrix} F(R_i, R_i) & F(R_i, R_j) \\ F(R_j, R_i) & F(R_j, R_j) \end{pmatrix}. \tag{3}$$

III. Local orientations are obtained from the tensor matrix through eigenvalue decomposition.

Local orientation can also be determined in the Fourier domain. The method fits a local window around the location of interest and the corresponding gradients are transformed into Fourier space and a structural tensor is calculated.

The structure tensor of image *f* is :

$$S = \int \omega\omega^T |F|^2 d\omega = \begin{pmatrix} S(1,1) & S(1,2) \\ S(2,1) & S(2,2) \end{pmatrix}. \tag{4}$$

$$S(i,j) = \int \omega_i \omega_j |F(\omega)|^2 d\omega. \tag{5}$$

To extract the local orientation these methods are applied to the following outcrop image. In well defined regions the orientations of the tensors are consistent with local geology.

Figure9:Greyscale folded outcrop for generating LVA (Source:http://gsfc.nasa.gov/Sect2/Sect2_1a.html).

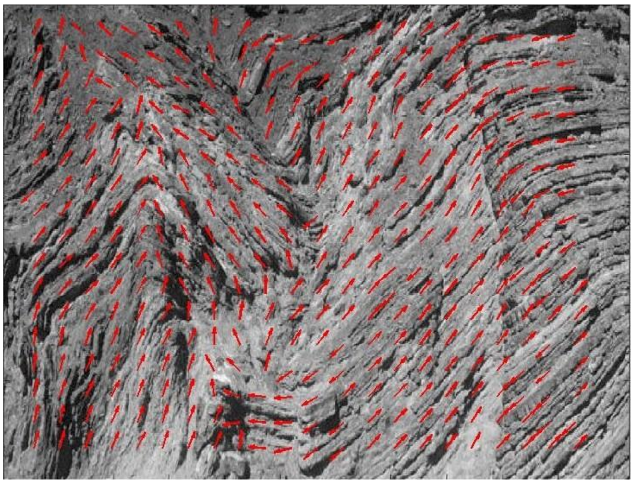

Figure 10: Using structure tensor in spatial domain to generate LVA for the outcrop

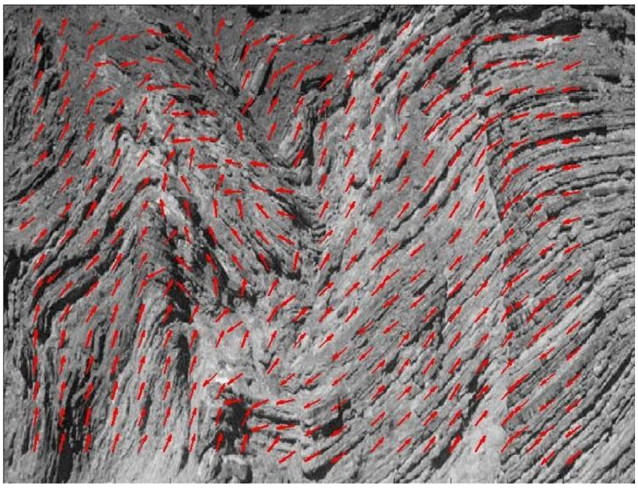

Figure 11: Using structure tensor in Fourier domain to generate LVA for the outcrop

4. Case Study 1 - Estimating Grade in a Porphyry Deposit

The first case study examines generating an LVA field for a copper/gold porphyry mine. This example was presented in Boisvert and Deutsch (2011), the focus here is on LVA field generation of a deposit with complex geology. The deposit is a typical cylindrical porphyry deposit with an unmineralized core. The LVA field is deemed to be tangential to a manually fit circle.

The 3D LVA field in this case can be considered as an amalgamation of 2D fields where the dip is considered to be 0° (vertical anisotropy is considered). The LVA field is fit by manually selecting the centroid of the deposit on plan slices and fitting a circle to the edge of the unmineralized zone. Within this circle the deposit is considered isotropic ($r_1=r_2=1.0$). Outside, the LVA field orientation is considered tangential to the circle. Finally, a transition zone is defined to reduce the artifacts between the isotropic unminearlized core and the highly anisotropic mineralized zone. The inner circle of the transition zone is assigned an anisotropy ratio of 1.0 and increases linearly to the outer boundary of the transition zone where the anisotropy is 10:1.

This is repeated for slices at 20m intervals. The center point of the circle (x and y coordinates) and the radius is estimated for each slice with inverse distance. This fully defines the orientation of the LVA field for each slice of the model. These 2D LVA fields are combined to generate the necessary 3D field.

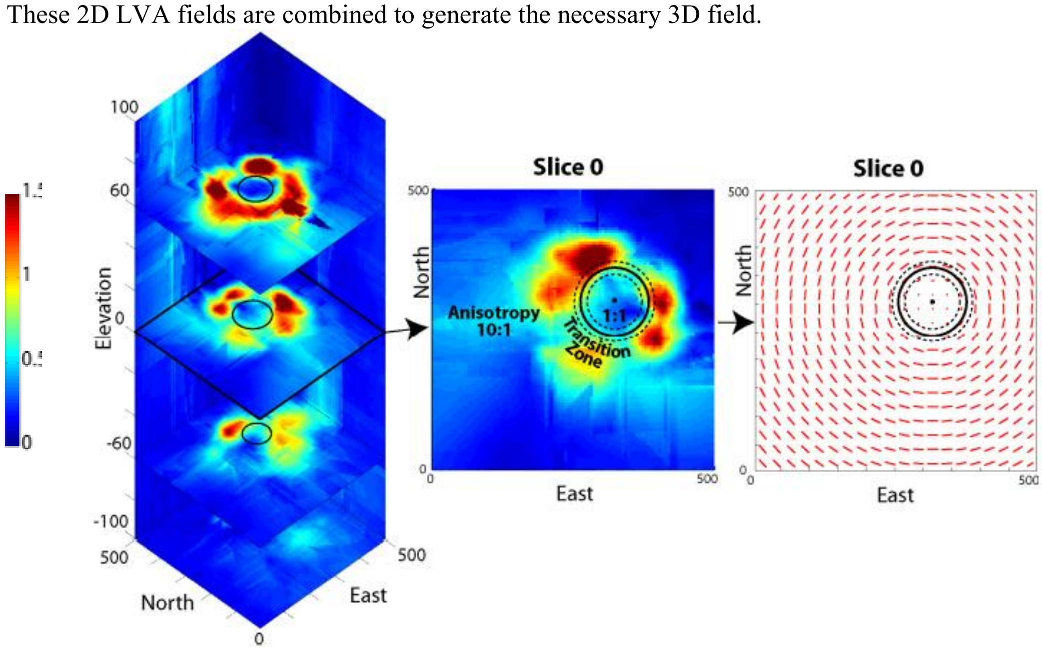

Figure 12: The colour scale shows the grade of the deposit. Left – Multiple slices of the block kriging map used to generate LVA field. Middle – For each slice the anisotropy ratio is defined manually; the ratio is 1:1 inside the circle and 10:1 outside. Right – LVA field for elevation zero. The length of the line is proportional to the anisotropy ratio (Boisvert and Deutsch, 2011).

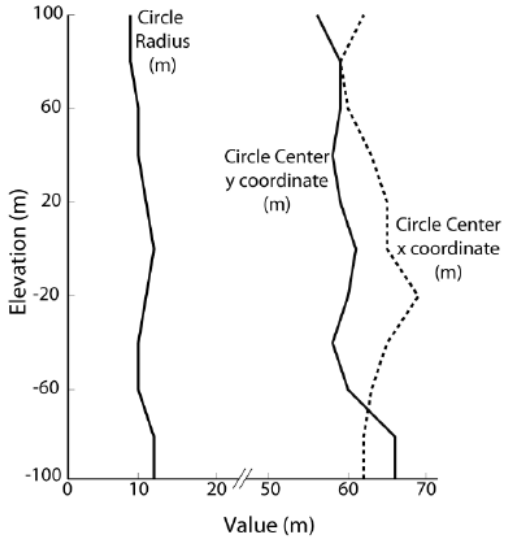

Figure 13: Parameters that define the isotropic core. The radius, X and Y coordinates of the circle are manually fit for every 20 m and interpolated in between (Boisvert,2010) .

5. Case Study 2- Uranium Role Front

Properties of interest (metal grade, porosity, etc) are often preferentially distributed along non-linear features of interest. Capturing these LVA features in numerical models can have a large impact on resource calculations as well as local planning. A uranium role front is described by a rotated parabola with grades preferentially distributed along the front. The LVA field is considered as the bounding surfaces of the role front (left and right on Figure 14). These are fit with polynomial functions and interpolated between cross sections to give the overall coarse geometry. The orientation of the LVA is tangential to the surface; between the surfaces the orientations are linearly interpolated to provide the exhaustive LVA field.

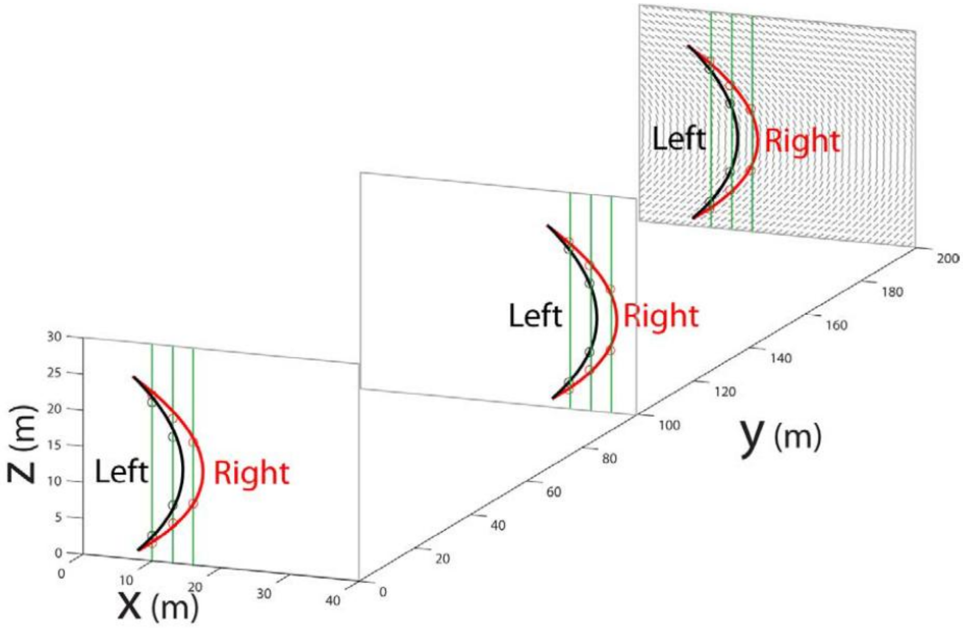

Figure 14: Schematic of the synthetic uranium role front deposit. The three defining sections and their surfaces (right and left) are generated from a total of 9 drill holes. The LVA orientation shown is on the 200 m cross section and is representative of the role front. The magnitude of anisotropy is assumed constant at 20:1 at the coarse scale.

6. Discussion and Conclusions

The goal of modeling with LVA is to generate realistic geostatistical models that capture important nonlinear geological features. Often when traditional stationary methods are applied the resulting linear features are deemed unrepresentative of the underlying geology.

Generating the required LVA field is difficult and case specific. Examples were presented to demonstrate a number of different techniques that could be applied to other deposits depending on local geometry. The governing factor in determining which technique to apply is the type of data available. LVA fields from exhaustive data are best assessed with a moving window technique while point data can be modeled by doubling angles. If the data is more qualitative or based on a geological interpretation of the deposit, the suggested approach is to fit polylines, splines or surfaces to the known features and use the slope (or dip) to define the LVA.

The main benefit of incorporating LVA into numerical modeling is to increase local accuracy. LVA fields can be used to model complex nonlinear geology and determine its effect on the variable of interest which can be critical for unbiased resource evaluation when insufficient data are available to control traditional techniques.

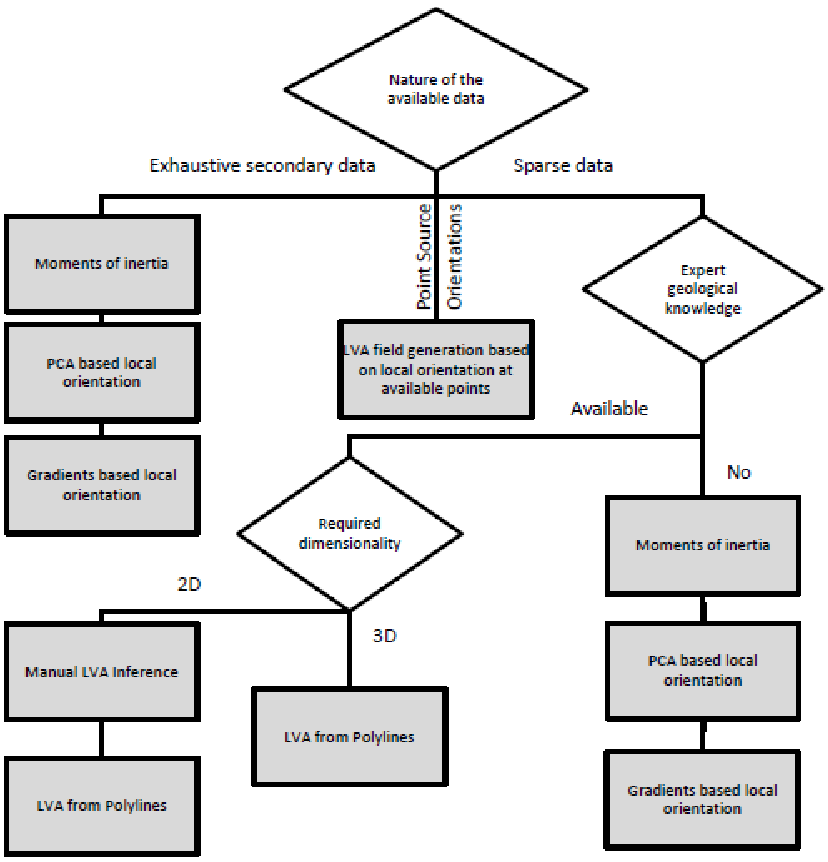

Figure 15: Decision tree for selecting a suitable LVA methodology.

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