On internal consistency, conditioning and models of uncertainty

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Abstract As the focus in spatial modeling has shifting towards generating multiple conditional realizations (posterior samples for Bayesians), issues have arisen regarding the nature of the uncertainty space spanned by these algorithms. In this paper, I propose a simple test to verify whether the space of these multiple realization is in fact consistent with the chosen theory by which the underlying algorithms were derived. Two examples of popular conditioning methods are investigated: sequential simulation and ensemble Kalman Filter (EnKF).

What is internal consistency?

In the statistical sciences, internal consistency is the extent to which tests or procedures assess the same characteristic, skill or quality. In the context of modeling uncertainty, I will therefore define internal consistency as the degree to which sampling methods honor the relationship between the unconditional model of uncertainty (prior) and conditional model of uncertainty (posterior) as specified under a (subjectively) chosen "theory" (for example: Bayes' rule). The "tests" performed are then various different ways of sampling from the same (conditional or unconditional) distributions. If these distributions are related to each other via a theory, then such "tests" should yield similar results. In this work, I apply various such tests on popular modeling techniques using Bayes' rule as the "theory"

Testing internal consistency

A simple test is designed for internal consistency between conditioning mechanism, theory and models of uncertainty. The test works as follows and applies to both Bayesian and frequentist views of modeling. We assume a forward model is given and has no uncertainty.

In the Bayesian context, the test is as follows

1. Sample a model $\mathbf{m}_{uc}$ from the prior $f(\mathbf{m})$

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2. Generate data **d** using the forward model **d**=g(**m**)
3. Sample a posterior model $\mathbf{m}_c$ from $f(\mathbf{m}|\mathbf{d})$
4. Repeat steps 1-3
5. Compare the set of $\mathbf{m}_c$ with the prior set of $\mathbf{m}_{uc}$

For the frequentist view, a similar reasoning is followed

1. Generate an realization $\mathbf{m}_{uc}$ using an unconditional algorithm
2. Generate **d** from **d**=g(**m**)
1. Generate a realization $\mathbf{m}_c$ using the conditional algorithm
2. Repeat steps 1-3
3. Compare the set of $\mathbf{m}_c$ with the prior set of $\mathbf{m}_{uc}$

What is the purpose? In repeating this workflow, we obtain multiple conditional models. The distribution of these conditional Earth model should be exactly the same as the unconditional or prior model, indeed, since we randomize the data in such a way that it is consistent with the prior, we should get back the prior:

$$\int_{\mathbf{d}} f(\mathbf{m}\,|\,\mathbf{d})\, f(\mathbf{d})\, d\mathbf{d} = \int_{\mathbf{d}} f(\mathbf{d}\,|\,\mathbf{m})\, f(\mathbf{m})\, d\mathbf{d} = f(\mathbf{m}) \int_{\mathbf{d}} f(\mathbf{d}\,|\,\mathbf{m})\, d\mathbf{d} = f(\mathbf{m})$$

Note that in this derivation we used the subjectively chosen “theory”, namely Bayes’ rule.

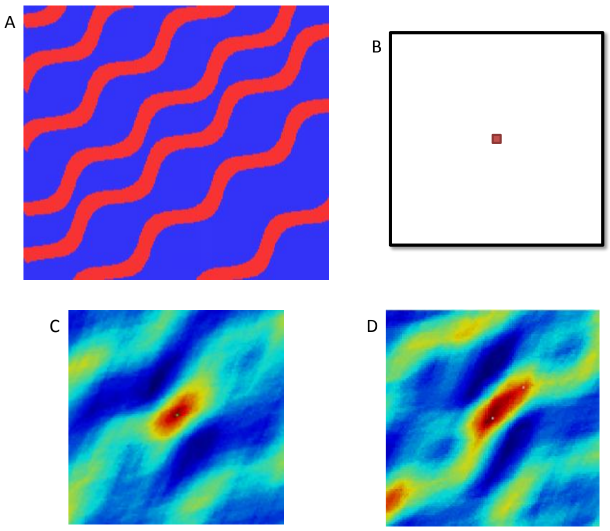

Figure 1: (A) training image, (B) single hard data location, (C) ensemble average of rejection sampler and (D) ensemble average with *snesim*

Example 1: conditional sequential simulation

Often, conditional sequential simulation algorithms treat hard data different from previously simulated nodes, and hence due to the difference between the conditional and unconditional algorithms, internal *in*consistency may be generated. Consider the simple example shown in Figure 1. A single hard conditioning data indicating channel is located in the center, see Figure 1B. A training image, Figure 1A, is given containing simple sinuous channels. Consider now conditioning first using a rejection sampler. 150 models are created that match the data. The rejection sampler uses the unconditional version of the algorithm, in this case *snesim* [3]. The ensemble average is provided in Figure 1C. Next, 150 models are created using the same *snesim* algorithm, but now the conditional version. Clearly the ensemble average in Figure 1D differs from the rejection sampler, meaning there exists an internal consistency problem between conditional and unconditional *snesim*. Why does this problem occur? Since *snesim* works with multi-grids, the single hard data needs to be relocated to the nearest coarse grid node. In case of reservoir modeling where the single well is a production well, this problem has considerable impact on flow predictions. A solution using a multi-resolution approach is presented in Honarkhah ([1], see also this conference).

Example 2: Ensemble Kalman filter

The formulation of the inverse problem from which the EnKF is derived requires a multi-Gaussian model for the variables and for the relationship between data and model variables to be linear. Consider an example inverse/conditioning problem in Figure 2. A simple injector and producer configuration is shown on a 31x31 grid. The data is the watercut over a period of 10 years. The prior is a set of realizations generated using a training image, see Figure 2C. To apply the EnKf method to these non-Gaussian fields and non-linear forward model, we use the metric ensemble Kalman filter [2], an adaptation of the ensemble Kalman filter performed after kernel transformation. Figure 3B shows the match of 30 posterior models. Consider now generating 30 history matched models using the rejection sampler. When comparing the conditional variance (CV) of the permeability fields generated using these two techniques, see Figure 3C,D, we notice the low variance of the ensemble Kalman filter as compared to the rejection sampler. Clearly, the linear and Gaussian hypothesis of the Kalman filter lead to internal inconsistency when applied to a non-linear and non-Gaussian problem (localization and increasing the ensemble does not solve the problem). In practice, this problem is hard to detect, since the models generated match the data and have similar patterns as the training image or prior models (see Figure 2E,D).

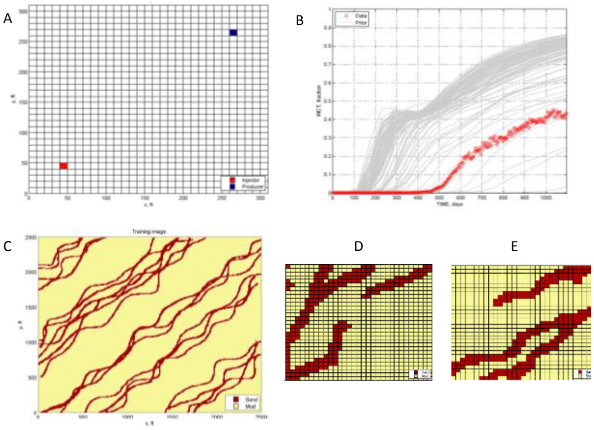

Figure 2: (A) location of injector and producer, (B) red=watercut data, grey = prior models, (C) training image (D) prior realization, (E) posterior metric EnKF realization.

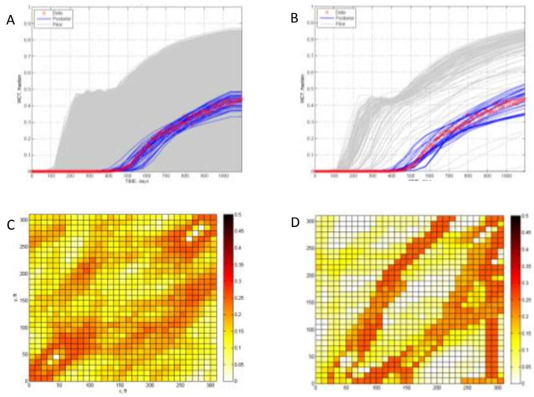

Figure 3: (A) results of rejection sampler (10.000 simulations (grey)), (B)) results of metric EnKF (30 simulations (grey)), (C) CV rejection sampler, (D) CV metric EnKF

Bibliography

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